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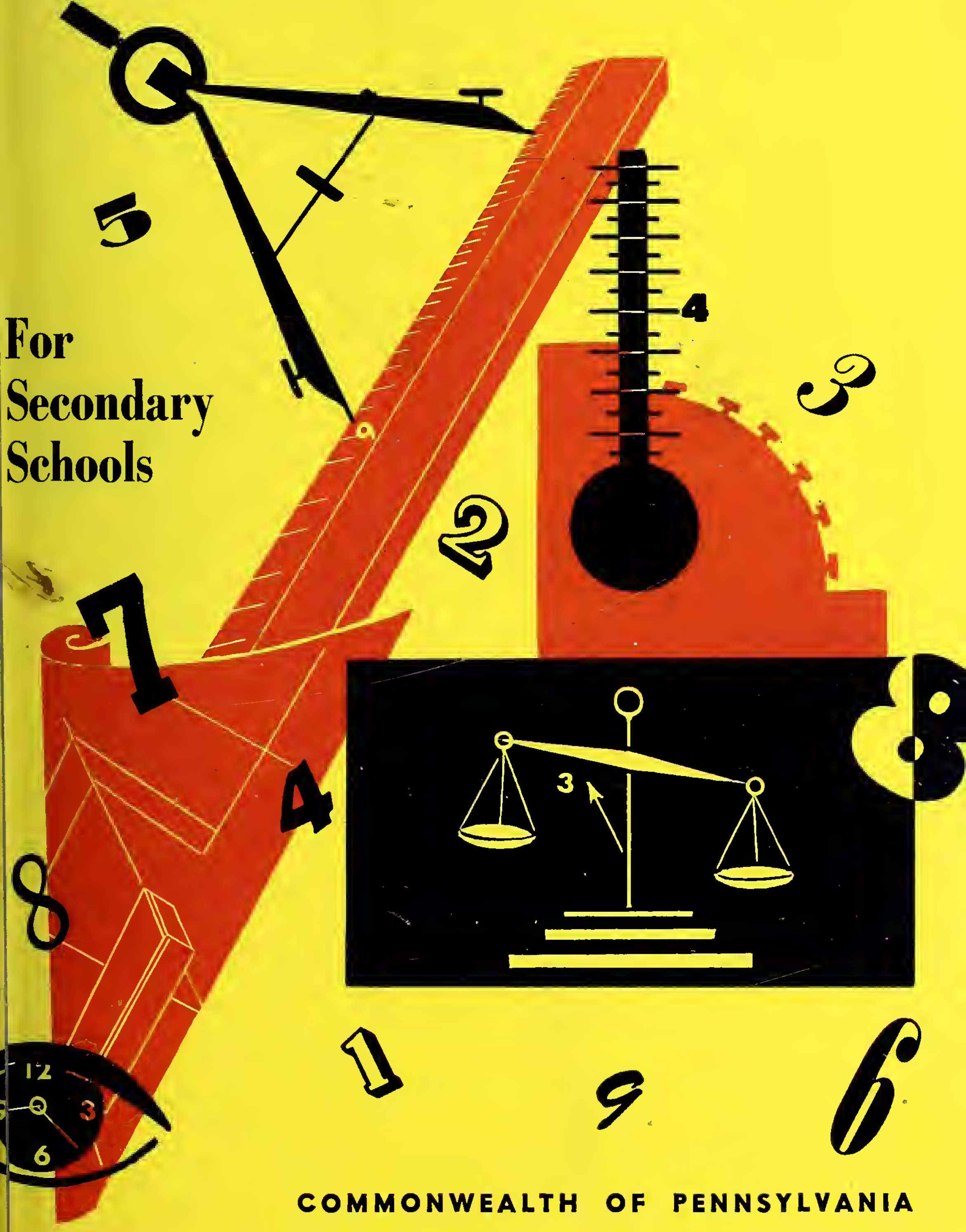
A Course of Study in
MATHEMATICS

For Secondary Schools



BULLETIN No. 360

COMMONWEALTH OF PENNSYLVANIA
DEPARTMENT OF PUBLIC INSTRUCTION
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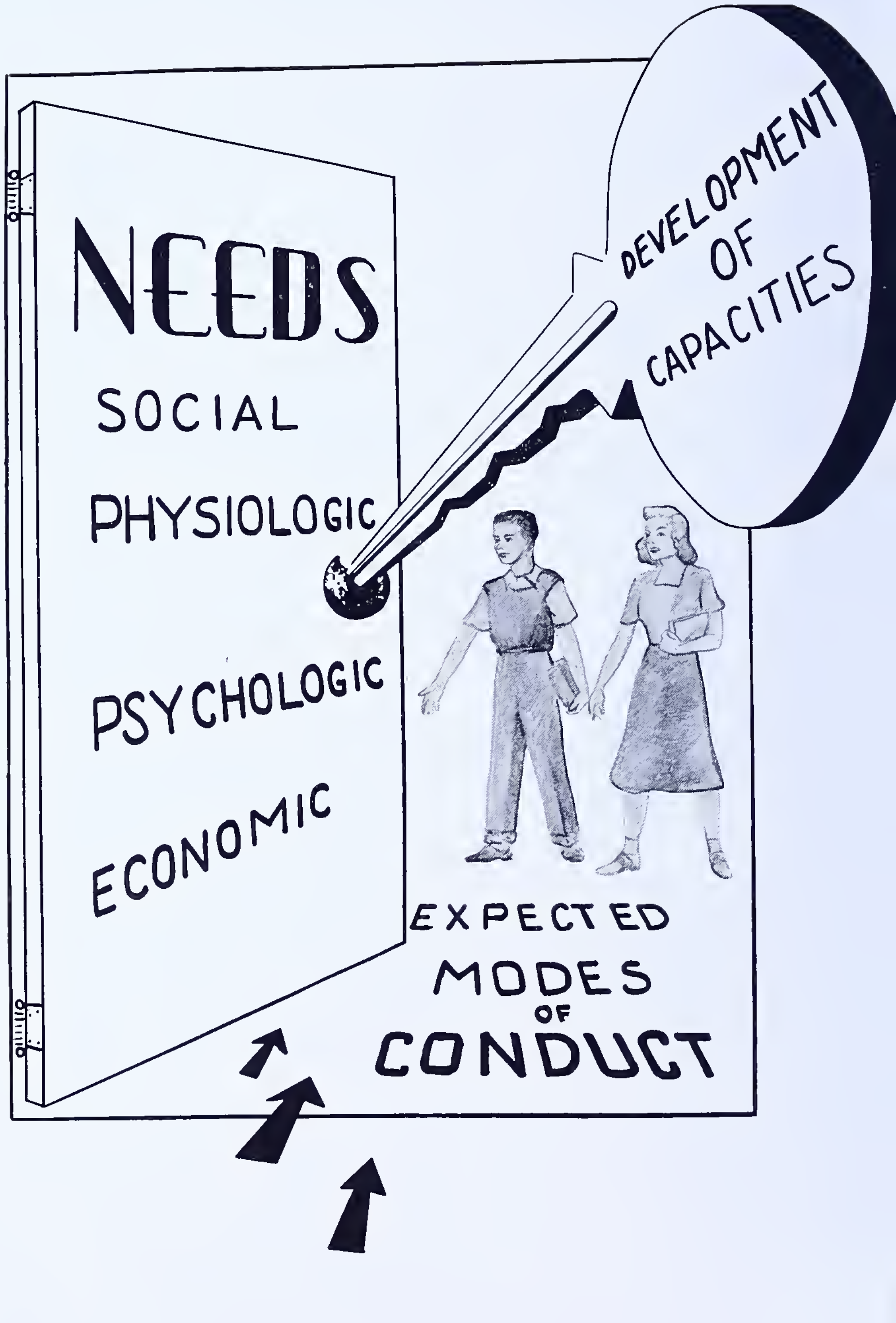
NEEDS
SOCIAL
PHYSIOLOGIC
PSYCHOLOGIC
ECONOMIC
DEVELOPMENT OF CAPACITIES
EXPECTED MODES OF CONDUCT

COURSE OF STUDY
in
MATHEMATICS
for
SECONDARY SCHOOLS
A Progress Report

BULLETIN 360
1952

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COMMONWEALTH OF PENNSYLVANIA
DEPARTMENT OF PUBLIC INSTRUCTION
HARRISBURG

matics—The Door to Opportunity

BULLETIN 360 is available for other than Pennsylvania public school distribution on receipt of check or money order for $1.25 made out to the Commonwealth of Pennsylvania and sent to the Editor, Department of Public Instruction, Box 911, Education Building, Harrisburg, Pennsylvania.

FOREWORD

MEASUREMENT has been indispensable in the process of civilization. The application of mathematics to problems of human progress is the basis of modern technology and improved standards of living. In this ever increasing development, the study of mathematics in the public schools must undergird the further specialized learning of the scientist, the engineer, the craftsman, and the technician.

In addition, the general applications of mathematics have essential values for the worker, the homemaker, and the citizen. The study of mathematics should promote for all youth the behaviors of thrift and consumer judgment which lead to financial security and social competence.

In this course of study provision is made for both special and general learnings. This present development should provide the basis for further curriculum enrichment so that the best possible results may be realized in the classrooms of our schools and in our communities.

Grateful appreciation is expressed to the members of the State Production Committee and to the members of the nine district committees whose cooperative efforts have created this Progress Report.

Norman Bucher, Mathematics Department, Senior High School, Lebanon; Ralph D. Horsman, Superintendent of Schools, Mount Lebanon; Catherine Lyons, Perry High School, Pittsburgh; Joy Mahachek, Head of the Mathematics Department at the State Teachers College, Indiana, Pennsylvania, have reviewed this manuscript.

The bulletin has been edited by Rachel S. Turner, Editor for the Department of Public Instruction.

Francis B. Haas

Superintendent of Public Instruction

March 1952

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INTRODUCTION

Purpose of Bulletin 360, Course of Study in Mathematics

To do the job better is a characteristic aim of democratic society. This applies with special force to the mainspring of that society—the agencies responsible for education. This bulletin has been developed to aid a special group in this educational program, those charged with that part of education known as "secondary mathematics education." No one person has a monopoly on how to teach better, but all can profit from the considered experience and the purposeful thinking of many teachers faced with the same challenge—the adequate preparation of youth for effective membership in a dynamic society.

At present, most of the secondary school programs in Pennsylvania are centered around organized fields of knowledge. To present the best in the racial heritage, is an accepted objective. The difficulty lies in the process of dealing with the time element—using the experience of the past to guide youth, who are concerned with the present, in order to prepare them for the future.

One of the major fields of organized knowledge is mathematics. It has become increasingly important in our rapidly changing society. Indeed, the Educational Policies Commission, in discussing the goal of self-realization, presented this challenge to the secondary school program. "In an age such as ours, where every phase of life is marked by applied science and technology, the appreciation and the use of basic mathematical skills offer significant assistance to self-realization."

How then can we as specialists in the field of mathematics do a better job of helping young people adjust to these problems both personal and social, now and in the future? This guide is the result of the cooperative efforts of mathematics teachers throughout Pennsylvania. It is not a mandate; it is essentially a record of suggested ways that have emerged from the thinking done by these teachers who have felt that they can make a better contribution to the total growth of adolescents by directing their efforts toward *"making mathematics meaningful."*

Relationship to Other State Bulletins

The Department of Public Instruction in Pennsylvania has produced Bulletin 233-B, *The Elementary Course of Study, An Interim Report.* Since a program of public education should be uniform in purpose and closely related in its various areas and levels, it is important that the mathematics program of secondary education continue the philosophy and build upon the foundation presented in the new elementary school program. To this end, the State Committee for Secondary School Mathematics studied the Arithmetic Section of 233-B and developed this guide in harmony with that phase of the elementary school program.

The State Superintendent of Public Instruction, Francis B. Haas, has stated, *"The major purpose of free public education in a democratic social order such as ours is to make the individual socially competent."* Bulletin 242, *Educating for Citizenship,* was written to give teachers help in finding ways to meet this objective. It is the intent of this guide in mathematics to give attention to the fact that all pupils must grow into an intelligent citizenry. What is the role of mathematics in meeting this social challenge? To the extent that answers can be suggested for this question, this guide is an elaboration upon Bulletin 242.

Relationship to Other Study Guides

This guide in mathematics is one of the subject area guides being developed simultaneously at the State level. Representativs from other subject areas—English, social studies, science, geography, and modern foreign languages—have met with the mathematics teachers in general conferences which were designed to help all groups toward common purposes.

Greater understanding of how we learn has given impetus to the trend in curriculum improvement toward unifying the educational program of the secondary school. It is one of the purposes of these bulletins to help schools to work toward broader areas of learning based upon the needs of the learners. This mathematics bulletin may be considered a source from which teachers may gain ideas concerned with those procedures and learning experiences which pertain to mathematical competence, knowledge, and understanding. There is also a section containing suggestions as to how teachers of mathematics may work more closely with their fellow faculty members in contributing to the total growth of each pupil.

How to Use This Guide

In using this manual, it is essential to understand Chapter I, "Bases for Curriculum Development," and Chapter II, "Improving Instruction," before considering individual courses and content. These courses are only a means toward the ends which are discussed in Chapters I and II. After that, one may properly appraise the course outlines. Learning experiences are important only in reference to their purposes.

The contents of this guide are the reflection of the practices and the ideals of your fellow teachers in Pennsylvania. Consider the material as suggestive. Where it harmonizes with your own teaching success, you may find incentive for further improvement. Where the approach may be different, you may be willing to experiment. Your contribution to curriculum improvement in Pennsylvania will come in recognizing that any curriculum requires constant change and improvement, and in evaluating the content and the methodology in terms of improved learning.

This is a Progress Report. This report is concerned more with broad purposes and directions than with detailed course outlines. It does not attempt to compete with or replace courses of study developed by local curriculum committees which have put in long periods of intensive study. State bulletins must be studied in the light of local needs and then adapted accordingly. Your comments and help in the later revision will be a source for continued improvement.

You may work with the other teachers in your school to develop ways in which your courses and theirs can become more functional and better articulated. Suggestions along this line are made in this bulletin. You may study your own pupils to find their general and special needs in order to:

1. Guide them into the proper courses outlined in this manual.
2. Better adapt your instruction to their needs.
3. Carry on continual evaluation to determine whether or not the learning is progressing in the direction of the stated purposes of the courses.

The Need for Curriculum Improvement in Mathematics

People who give serious thought to curriculum improvement recognize that it is a never-ending process. Changing philosophies and objectives have put so many demands upon the secondary school program that there is a tendency to have a cluttered curriculum in that we tend to add more content than we are willing to drop. It becomes necessary, then, for some agency to appraise the situation and to ask what are the educational values of mathematics in secondary schools today.

The Commonwealth, by pooling the best thinking and practices of its teachers, is in a strategic position to produce a broad outline of a desirable educational pattern. Teachers find security in turning to some source other than their immediate means for guidance.

In addition, educators in general and mathematics teachers in particular have become increasingly dissatisfied with the effectiveness of the mathematics program and have voiced concern about such problems as:

1. Why is there such a large number of failures in mathematics?
2. What is the reason for the lack of practical applications?
3. Why is there a tendency to reduce the mathematical training of the average pupil?
4. Why do pupils tend to dislike mathematics?
5. Why is content so largely concerned with the abstract and the mechanical?
6. What is the need to have a more unified program?
7. How can instructional aids and materials be used more effectively? Where can we get them?
8. Why is there lack of fundamental preparation at all grade levels?
9. Why is there lack of retention of the materials of mathematics?

10. Are there enough provisions for the varied needs of all pupils?
11. Is there too little coordination of methods used in presenting similar topics by successive teachers?
12. Is there insufficient guidance of various types?
13. How can instruction be individualized?
14. How can meaningful teaching be developed?
15. How can different types of outcomes be evaluated?
16. How can the interest and effort of many pupils be aroused?

Overview

The material in the succeeding chapters of this manual is an attempt to give suggestions in the areas posed by these and similar problems facing teachers today. In high light, these chapters are as follows:

Chapter I: *Bases for Curriculum Development in Mathematics.* A discussion of the points of view and the objectives underlying the suggested program. Suggestions for improved articulation.

Chapter II: *Improving Instruction.* A discussion of common instructional problems with suggestions as to how the learning may become more effective.

Chapter III: *Program of Studies: Scope and Sequence.* Essentially a more detailed discussion of the courses listed in Chapter II. Presentation of individual course objectives and suitable learning topics and experiences.

Chapter IV: *Evaluation.* A consideration of how the total program may be appraised and revised. Methods of discovering and measuring pupil progress.

Chapter V: *Supplementary Materials.*

Evaluation of Bulletin 360

In order that there may be an evaluation of this bulletin, the check list on the following pages is provided. It is requested that this be removed or reproduced and checked, after the course of study has been used for a semester, and that it be given or sent to your district chairman whose name appears on page vi.

REPORT OF REACTION TO BULLETIN 360
COURSE OF STUDY IN MATHEMATICS FOR THE SECONDARY SCHOOLS

A Progress Report

NOTE: The user of this bulletin is requested to fill out this report, after a semester of use, and to send it to the district chairman of the mathematics committee in his area.

HAVE THE FOLLOWING BEEN OF PRACTICAL USE TO YOU?

Chapter I. Bases for Curriculum Improvement

The Needs of Youth .... Yes.......... No..........
Purposes of Mathematics Education .... Yes.......... No..........
Articulation .... Yes.......... No..........
Enrichment .... Yes.......... No..........
Provision for the Below Average Student .... Yes.......... No..........
Some Do's and Don'ts .... Yes.......... No..........
Making Mathematics Meaningful .... Yes.......... No..........

Chapter II. Improving Instruction

Discovering for One's Self in the Classroom .... Yes.......... No..........
Improving Study .... Yes.......... No..........
A Laboratory Approach to Mathematics .... Yes.......... No..........
Development of Units .... Yes.......... No..........
Individualizing Instruction .... Yes.......... No..........
Using Community Resources .... Yes.......... No..........
Providing Guidance .... Yes.......... No..........

Chapter III. Program of Studies: Scope and Sequence

Introduction—A Master Plan .... Yes.......... No..........
Mathematics in Grades Seven and Eight .... Yes.......... No..........
Common Learnings Program:
  Basic Mathematics I .... Yes.......... No..........
  Basic Mathematics II .... Yes.......... No..........
  Basic Mathematics III .... Yes.......... No..........
  Basic Mathematics IV .... Yes.......... No..........
Specialized Mathematics:
  Algebra I .... Yes.......... No..........
  Algebra II .... Yes.......... No..........
  Geometry .... Yes.......... No..........
  Grade Twelve Mathematics .... Yes.......... No..........

Chapter IV. Evaluation in Mathematics

Purposes of Evaluation .... Yes.......... No..........
Outcomes Which Are to Be Expected .... Yes.......... No..........
Methods for Evaluating Outcomes .... Yes.......... No..........

Chapter V. Supplemental Materials Yes.............. No..............

Appendix

Results of An Inquiry On the Needs of Youth ....Yes.............. No..............
Partial Listing of Some Vocations Which Require Mathematics.Yes.............. No..............

What constructive suggestions can you make for future deletion, inclusion, or enrichment in the above chapters?

Chapter I ....
Chapter II ....
Chapter III ....
Chapter IV ....
Chapter V ....

What is your general opinion of the bulletin?

Signed

Course of Study in Mathematics for Secondary Schools

CHAPTER I

BASES FOR CURRICULUM IMPROVEMENT

There are several approaches to curriculum development. This Progress Report is based essentially upon the pupil-needs approach. In the preliminary stages of its preparation, consideration was given to the secondary school pupil, what he is like, and what the school is attempting to do for him. After a consideration of these factors, the key question seemed to be, What can mathematics do to contribute to the broadening purposes of secondary education?

The outlines in Chapters II and III are based on the fact that mathematics has a guidance contribution in addition to the weight of its matchless tradition.

Two types of needs are found: general and special. The first group is in response to those demands which are made upon all youth because of the nature of our present society and because of the characteristics common to most adolescents. An investigator would expect to find these needs in all types of communities, in all sections of the country. What would differ would be the degree of the need rather than the lack or presence of that factor. That is, in some localities, health needs might be more pressing than in others, but the maintenance of good mental and physical health should be of concern to all schools. The special needs grow out of the ways in which adolescents differ—their backgrounds, their native abilities, their interests, their plans for the future. If a mathematics program is to be an example of good curriculum practices, it must consider both common and individual needs.

There have been numerous studies of the needs of youth, but for purposes of agreement the so-called "Ten Imperative Needs of Youth"[1] *were selected as a starting point.* The following outline was developed to answer the question of whether or not mathematics education does play an important part in the total plan.

---

[1] *The Bulletin*, National Association of Secondary-School Principals, Volume 31, Number 145, March 1947; Volume 32, Number 154, April 1948; Volume 33, Number 164, October 1949.

THE NEEDS OF YOUTH

General Needs of Youth	Examples of Contributions of Mathematics
1. He needs skills and those understandings and attitudes that make the worker an intelligent and productive participant in economic life.	1. Mathematics as applied to shop problems, vocational occupations, business offices, etc.
2. He needs physical fitness and good health.	2. Height and weight, insurance, recipes, calories, etc.
3. He needs to understand the rights and duties of the citizen in our democratic heritage; to perform with diligence and competence his obligations to the state, nation, and world.	3. a. Cultural aspects—the history of numbers b. Taxes—buying and consuming, economic aspects, social security
4. He needs to understand the importance of family relations in the social institutions. A happy well-adjusted family is the basis for a healthy community.	4. Budgeting, accounting, insurance
5. He needs to know how to spend his money wisely and the economic consequence of his actions.	5. Economic estimates, buying and selling Currency exchange in foreign travel
6. He needs to understand scientific methods and the influence of science on human life.	6. Statistical comparisons, graphs, skill in the use of instruments, space perception, history of mathematics
7. He needs to develop tastes for the finer things in life, such as literature, music, art, and nature.	7. Geometrical forms in nature, art, architecture
8. He needs to develop respect for the other person and in turn discover the ethical values and principles that enable society to live and work peacefully.	8. Order, law, rhythm and symmetry of all mathematical relationships
9. He needs to develop the habit of rational thinking and to express his thoughts clearly and with understanding.	9. Logical forms: Inductive and deductive reasoning, indirect proofs, critical analysis, etc. (This is the chief field of mathematics.)
10. He needs to be able to use his leisure time effectively and wisely.	10. Thought-provoking instruments Puzzles, games, etc. Leisure-time ideas in units of learning

How well are these needs being met by the present program of secondary education? How can education in mathematics contribute more effectively to their realization? The results of the questionnaire shown in Appendix 1 indicate areas in which both serious lags and opportunities exist.

The Adolescent

There are several ways in which we may learn about adolescents. The best way is to study the particular adolescents in our classes. What adolescents do,

what they say, and what they write will provide information we need if we are going to teach our "object" as well as our "subject."

No adolescent was born a liar, a cheat, or a thief. A pattern of factors or pressures may make him into a liar, or a cheat, or a thief. Adults may help to make him what he is. If we are willing to study the adolescent, if we focus our attention on an adolescent's virtues, if we can provide for the adolescent's basic human needs while we try to locate the causes of his difficulty, we can increase our ability to produce desirable citizens.

A realization that behavior is caused by a pattern of pressures from the culture in which a person is reared, from the peer group (the class), from the physical being, from the emotional being, and from maturing, helps us determine the factors causing a particular behavior. Our use of activity, books, and language may eliminate the pressures causing undesirable behavior. There is no *one* way to help every adolescent. Our procedure will vary with each adolescent. The peer group may do more for those needing help than we can, if we will only let it.

We also learn about adolescents by studying report of authorities. From these reports, we learn that pupils of the junior and senior high schools are generally grouped as follows:

Preadolescents	10-12 years
Early Adolescents	13-16 years
Late Adolescents	17-21 years

Generally, boys mature slightly later than girls. Within the sex groups, marked individual differences in maturation may be found. There is also a close relationship between a pupil's behavior and the level of his sexual development.

The Needs of Youth

No matter what the stage of development, every pupil needs:

1. *Security*—a sense of belonging—a sense of being accepted by teacher and fellow pupils
2. *Affection*—a feeling that his teacher likes him
3. *Recognition*—a feeling that his teacher knows his strengths, achievements, and interests as well as his weaknesses, failures, and problems
4. *Success*—a feeling that he can attain goals that he, his classmates, and his teachers set
5. *Freedom*—a feeling that permits him to express his thoughts without fear and to do what he wants, provided that what he does is not detrimental to others

In the light of these basic human needs, we might consider:

1. The relation of failing grades to security
2. The relation of red ink to success
3. The relation of assigned reading or writing to freedom
4. The relation of sarcasm to affection
5. The relation of chilly formality to recognition

Junior High School Pupils

To work effectively with junior high school pupils, we should know that these young people

1. Have a strong desire to be members of groups, gangs, or clubs. They often like to develop constitutions, bylaws, and formalities for their organizations
2. Value peer-group approval or criticism more than that of adults
3. Often have a relatively poor physical coordination. Boys, especially, at this level may hate to "get up before the class" or to dance
4. Prefer group to individual games and activities
5. Have wide intellectual interests
6. Like variety, adventure, and excitement. Girls often become interested in romantic love
7. Tend to be self-conscious
8. Are interested in sex, but try to conceal the interest from parents and teachers
9. Fluctuate between dependence upon adults and an independence that may be rebellious
10. Accept or reject parents and home
11. Are strongly idealistic. Hero worship is common, and evaluations are made as all good or all bad—all right or all wrong.
12. Are often socially insecure and may compensate by making themselves conspicuous in dress or behavior
13. Are interested in the opposite sex, but generally go around with members of their own sex groups. They may have crushes on age-mates or adults; their choices of "best friends" shift rapidly
14. Are greatly interested in money-making activities
15. Clarify their sex roles—boys want to act like and be recognized as men; girls, as women. The "tomboy," however, retains status with both sexes of her age group. The "sissy" loses status in his age group

16. May be able to read as well as an adult but do not necessarily enjoy reading or understand adult materials. These pupils may be avid readers of comics
17. Often need help in accepting their personal appearance and making the most of it. Youngsters who are excessively fat or thin—youngsters with freckles, red hair, or pigeon toes—may require special help

Senior High School Pupils

1. Have three important problems to solve:
    - a. Choosing a life work
    - b. Becoming independent of parents and adults
    - c. Assuming adult sex role
2. Pair off with members of the opposite sex in addition to going around in crowds
3. Conform to group standards in dress and behavior. Fads, creative vocabularies, and conspicuous behaviors are common
4. Often cover shyness and sensitiveness with a "don't care" attitude
5. Resent adult domination
6. Are deeply idealistic
7. Are so preoccupied with activities, pleasures, and friends that all reading to appeal must be extremely meaningful
8. Are interested in the concrete rather than in the abstract. The relation of the past to the present is interesting to them
9. Show widening and deepening of intellectual and aesthetic interests
10. Are interested in learning about the ways of the world and are curious about human relationships
11. Differ in general interests. Boys are concerned with athletics and work. They prefer books of real adventure, science, athletics, and mysteries. Girls prefer romance and current best sellers
12. Differ according to sex in oral and written composition interests. Boys tend to avoid writing, preferring to talk. If they must write, they prefer to write about personal experiences. Girls like to write letters about friends, dates, home or school life
13. Often have one or more hobbies which make a good basis for classwork
14. May feel awkward and embarrassed because of size, physical features, skin eruptions, and body odors
15. Often excel adults in physical coordination and dexterity

16. May become interested in a study of the effect of smoking and drinking on health, even while using the practices as symbols of revolt or sophistication
17. Often want a chance to write about or discuss personal problems and concerns
18. Begin to build personal philosophies and to express opinions on religion, peace, economic systems, intergroup relations, government, and ethics
19. Continue to mature in ability to reason. May demonstrate an unusual quality and depth in abstract thinking and problem-solving if permitted or encouraged
20. Usually believe that human life can be bettered and are eager to make it better

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Havighurst, Robert, and Taba, Hilda, *Adolescent Character and Personality.* Ne York, John Wiley & Sons, Inc., 1949

Hurlock, Elizabeth, *Adolescent Development.* New York, McGraw-Hill Book Compan Inc., 1949

National Society for the Study of Education, *Adolescence.* Forty-third Yearboo Part I. Chicago, University of Chicago, Department of Education, 1944

Tryon, Caroline, *Evaluations of Adolescent Personality by Adolescents.* Washingto National Research Council, 1939

Zachry, Caroline, and Lighty, Margaret, *Emotion and Conduct in Adolescence.* Ne York, Appleton-Century-Crofts, Inc., 1940

Fiction also contributes to an understanding of an adolescent

Bright, Robert, *The Life and Death of Little Jo.* New York, Doubleday & Compan Inc., 1944

Dreiser, Theodore, *An American Tragedy.* Cleveland, World Publishing Compan 1947

Farrell, James, *Father and Son.* Cleveland, World Publishing Co., 1940

Maugham, Somerset, *Of Human Bondage.* New York, Blue Ribbon Books, Inc., 1939

Maxwell, William, *The Folded Leaf.* New York, Harper and Brothers, 1945

Motley, Willard, *Knock On Any Door.* New York, Doubleday and Company, Inc., 19

Rawlings, Marjorie, *The Yearling.* New York, Modern Library, Inc., 1938

Saroyan, William, *My Name Is Aram.* New York, Harcourt, Brace and Compan Inc., 1937

————, *The Human Comedy.* New York, Harcourt, Brace and Company, In 1943

Sinclair, Jo, *Wasteland.* New York, Harper and Brothers, 1946
Smith, Betty, *A Tree Grows in Brooklyn.* New York, Harper and Brothers, 1943
Tarkington, Booth, *Alice Adams.* New York, Doubleday and Company, Inc., 1921
Walker, Mildred, *Winter Wheat.* New York, Harcourt, Brace and Company, Inc., 1944
Wolfe, Thomas, *Look Homeward, Angel.* New York, Grosset and Dunlap, Inc., 1939

Other sources of information on adolescents include radio, television, plays, poems, and films. The following are representative films that deal with adolescents and their problems:

A Criminal Is Born
Feeling of Hostility
Feeling of Rejection
Learning to Understand Children (Parts I and II)
Make Way for Youth
Shy Guy

and many others. These films are available through rental from your nearest film library.

PURPOSES OF MATHEMATICS EDUCATION

Point of View

To work toward a common goal, guiding statements or points of view should be formulated. In view of the purpose of secondary education and the understanding of the role of mathematics in that education, the following statement is a reflection of the points of view behind this bulletin:

> **A good mathematics program should grow out of and contribute toward the broad purpose of secondary education. Mathematics should present suitable learning experiences for all youth throughout the entire secondary school. It should be concerned with all phases of pupil growth. The mathematical program should concern itself with the common needs of youth and with the special needs of individuals. This program should foster an awareness of relationships, not only with the preceding and the following courses in mathematics, but also with other fields of learning.**

Learning experiences and materials in mathematics should be selected with due consideration to their contribution toward the attainment of desirable pupil growth. The following are goals for pupil development and therefore are guideposts for teachers in helping secondary school pupils to:

1. Develop those mathematical behaviors which will better fit pupils for useful service to their community and for ethical living
2. Gain an appreciation of the importance and power of mathematics in the development of societies—past, present, and future
3. Discover vocational possibilities which open if pupils attain certain competencies in mathematics
4. Develop the ability to analyze and to solve problems of living
5. Recognize the various relationships within the field of mathematics
6. Recognize the relationships of mathematics to other fields of learning

7. Develop an understanding of concepts of time, space, and quantity
8. Build foundations which will permit the study of higher levels of mathematics
9. Analyze themselves as to their abilities to deal with quantitative relationships
10. Develop proficiency in mathematics as a method of communication
11. Establish desirable work habits and study skills

Ways toward a better implementation of these purposes are discussed in the following sections:

ARTICULATION
ENRICHMENT
PROVISION FOR THE BELOW-AVERAGE STUDENT
SOME DO'S AND DON'TS
MAKING MATHEMATICS MEANINGFUL

ARTICULATION

Articulation, correlation, and *integration* are three terms which have moved from halls of educational theory into the classrooms of the secondary school. As yet, however, there is much to be done in working out ways and means to bring greater unity into the learning experiences of the pupils. Basically, the three terms that have been mentioned challenge teachers to give more attention to establishing relationships both in "vertical articulation" and in "horizontal articulation."

Vertical Articulation

The secondary school is but one phase of education for living, and a such should provide an orderly sequence that will enable the pupil t maintain steady growth. As was pointed out in the Introduction, it i the responsibility of the secondary school teacher to know and to understand the philosophy, the objectives, the course content, and the learnin techniques of the earlier grades as set forth in Bulletin 233-B.

In like manner, teachers must have knowledge and familiarity wit the work of the succeeding grades and with higher education. In som areas formal college entrance requirements still hamper experimentatio but this is one place where secondary and college people need to ge together to work out ways of meeting their common purposes. Numerou investigations and educational experiments such as the Eight-Year Stud and the current Michigan Agreement[1] cast considerable doubt on th validity of the so-called college preparatory subjects for determinin success in college, especially those of the liberal arts type.

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[1] For an outline of the Michigan Agreement see *Bulletin of the National Association of Secondar School Principals* for April, 1949.

Some teachers, in their concern for entrance hurdles, overlook the fact that even those college-bound are eventually to enter life as workers, as homemakers, as adult citizens. Either the Common Learnings Program or the Specialized Program as outlined in Chapter III will adequately prepare a person to be successful in college as far as mathematical competence is concerned. The essential difference is that the Common Learnings Program should be used for those who do not intend to pursue mathematics courses in college.

Within the field of mathematics, teachers should learn how best to correlate the various parts. How and where can algebra be correlated with arithmetic? How and where can arithmetic and algebra be correlated with geometry? How and where can algebra and geometry be correlated with trigonometry? Indeed, one of the charges leveled against the traditional teaching of mathematics is that there has been too much of a tendency to keep the various years in "water-tight compartments." Noted mathematicians, such as W. D. Reeve, William Betz, C. N. Shuster, and D. E. Smith, have urged that less emphasis be placed on algebra and geometry and trigonometry as specific subjects. The recommendations with the course outlines which are presented in Chapter III call attention to ways in which the barriers can be broken down. The Basic Mathematics itself is one example, as is Geometry, where both plane and solid geometry are brought together in their natural relationships.

Horizontal Articulation

Attention must be paid also to the relation of mathematics to other subjects that the pupil is studying during the school year. While the bond between mathematics and basic sciences—such as physics, chemistry, astronomy, biology, and geography—is more manifest than the relationships with other fields of study in the secondary school, there are opportunities in almost every category.

It is important that there be close cooperation among members of the teaching staff so that the correlation be most effective. For example, the mathematics teacher might well emphasize graphs when the pupil needs to interpret them in his geography or his social science classes. Here is an opportunity for a faculty to work together on the treatment of life problems, to utilize suggestions in the guides to foster local curriculum development.

Following are some suggested areas of cooperation for mathematics and other subject areas:

I. Art

Concerning the integration of art with other subjects, George T. Miller, Chief, Art Education, Department of Public Instruction, presents this comment:

In the Curriculum Bulletin for September, 1949, a reference is made to *fostering creativity* on the part of the student as one of the items of better curriculum building. This participation is close to the underlying philosophy held by us in Art Education.

I believe that there are activities, problems, plans, lessons, or projects in mathematics that will greatly enrich the learning experience through offering original and applied solutions and participation in these activities on the part of the student. I am thinking of such activities as are akin to art education in its wide and inclusive range that will make, through clearer graphical thinking, improvement in the curriculum. Since all of us are largely visual-minded, art becomes a universal language that may be utilized to serve, enrich, and integrate learning in all subjects.

II. Geography

A workable knowledge of mathematics is needed and desirable for a better understanding of the following problems in geography:

1. Practical uses of weights and measures of various kinds:
   Short and long ton
   Cords of wood
   Gallons
   Bushels
   Barrels of various materials
   Bales by volume and bales by weight
   Metric system
   Horsepower
   Per capita per square mile
2. Problems dealing with the globe—angles, degrees, circumference diameter, proportion, fractions, percentage, map scales, great circle meridians, and parallels
3. Interpretation of time belts and time at various parts of the earth
4. A study of weather and climatic includes problems dealing with
   Changing of Fahrenheit to centigrade
   Isobars, isotherms, and isallobars
   Relation of pressure in inches to pressure in millibars
   Relation of wind velocity to pressure gradient
   Relative humidity expressed in the form of a ratio, fraction, o percentage
   Weight and pressure
   Calibration of thermometers and altimeters
   Distribution of gases at different altitudes
   Temperature inversions and lapse rates
   Mathematical representation of climatic data
5. Population per square mile when area and population are give
6. Various uses of graphical presentation, such as vertical and hor zontal, bar, dot, circle, and percentage graphs. Not only an unde

standing of graphical representation but also practice in figuring production of various articles and products, and construction of accurate graphs to show what has been found or produced

7. Problems dealing with the movements of the earth, moon, and planets in space, and the various changes which occur because of the inclination of the earth's axis
8. Problems dealing with the duration of daylight, an understanding of which could include a workable knowledge of algebra and plane geometry
9. Problems dealing with minutes and seconds when studying latitude and longitude, and longitude and time
10. The construction and interpretation of various map projections. For a working knowledge of the earth's grid some spherical geometry and trigonometry are needed.

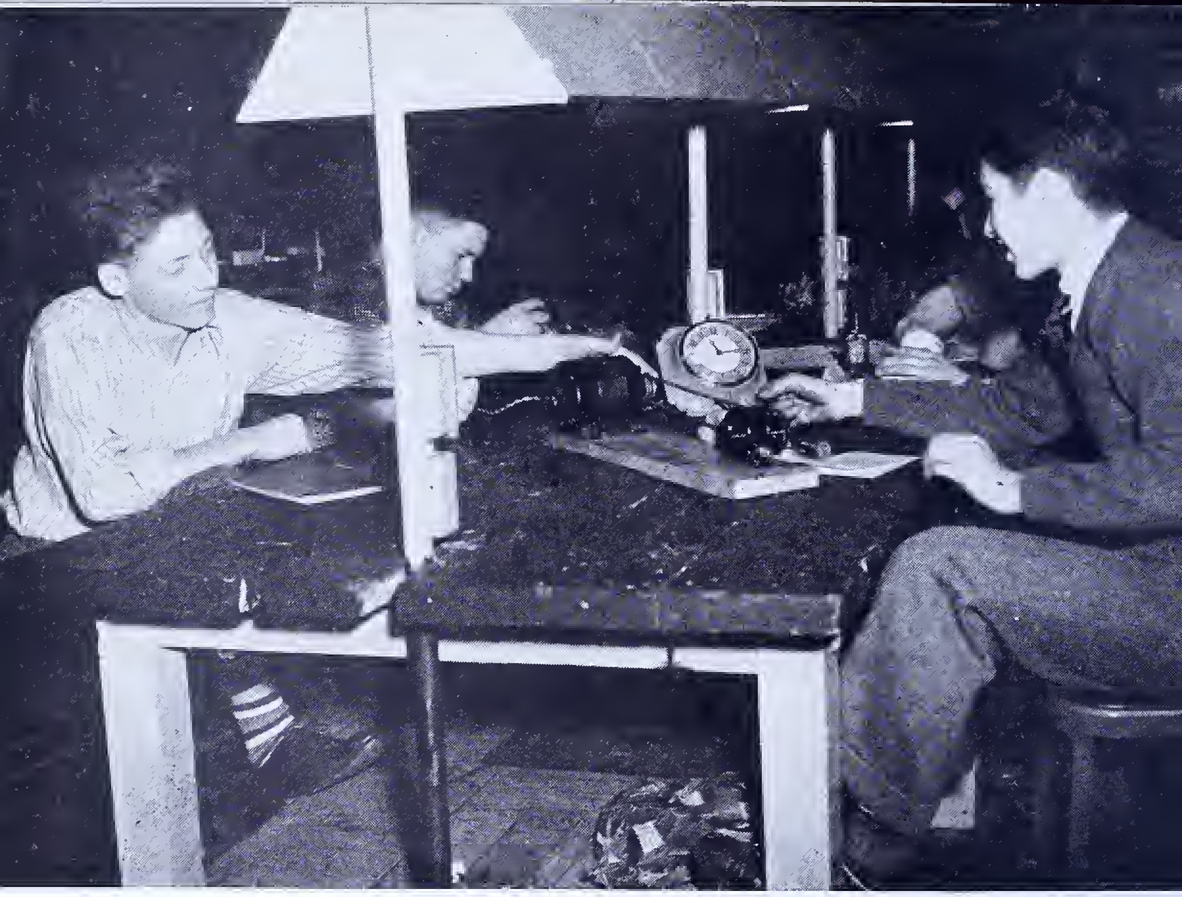
The above problems will serve to show that there is a definite relation between geography and mathematics. With modern applications of geography to the global idea of thinking, this has become more evident.

The relation of mathematics to geography not only needs to be taught but it may also serve to motivate the study of both subjects.

III. English

Desirable mathematical knowledge for pupils in English would include:

1. An understanding of the symbols and the processes of mathematics based upon actual experience, followed by pictures and then by symbols in speech and in writing
2. Experience with the everyday personal use of mathematics, as in keeping bank accounts, keeping budgets, economy of marketing, etc.
3. Ability to speak, to read, and to write accurately the language of mathematics
4. Ability to think and to reason accurately both deductively and inductively. Also the ability to sort the pertinent from the extraneous data and the false from the real
5. Some understanding of and appreciation for the contribution of mathematics to present world culture
6. Understanding of the basic concepts involved in statistics and their function in society
7. Ability to write, to read, and to interpret maps, charts, and graphs
8. Development of the spirit of inquiry, with knowledge of how to move from curiosity and inquiry to thought, to action, to completion, to evaluation
9. Knowledge of and ability in the reading and study skills involved in mathematics



SCIENCE AND MATHEMATICS

IV. Foreign Language

Desirable knowledge for pupils in foreign language study would include:

1. Some knowledge, at least the names, of the great mathematicians of all countries
2. Whenever and wherever possible, students of foreign languages who are in the mathematics class should be asked to find and report on the etymology of mathematical terms. Since these words show the functional origin of the terms, such a study should clarify and explain the basic meaning
3. German and French publications on mathematics have been very important. These journals, books, etc., in foreign languages might be made available for examination by the pupils
4. All pupils should know more about the metric system

V. Science

Mathematics is a tool for studying science

1. Training in the scientific methods: inductive and deductive reasoning. Emphasis upon relationship between assumptions and conclusions

2. Ability to handle intelligently such mathematical skills and concepts as: equations, conversion from or to the metric system, principles of variability, substitution into formulas, the law of proportions, decimals, approximation, finding lowest common denominator, squares and square roots, forming equations from verbal descriptions, graphs and charts, vectors

VI. Social Studies

Desirable knowledge for mathematics in social studies would include:

1. Testing of conclusions
2. Interpretation of graphs and statistical data
3. Understanding of general economic conditions
4. Uses of mathematics in history—land values and geometry in the valley of the Nile
5. Taxation
6. Population studies
7. Projects in collecting and analyzing data
8. Critical and analytical thinking underlying social problems
   a. Administration and interpretation of local laws
   b. Validity of arguments presented, propaganda, misleading advertisements, etc.
9. Economics—home finances:
   a. Budgeting income
   b. Keeping accounts
   c. Checking bills
   d. Installment buying
   e. Buying and owning a house or a car
   f. Taxes and insurance

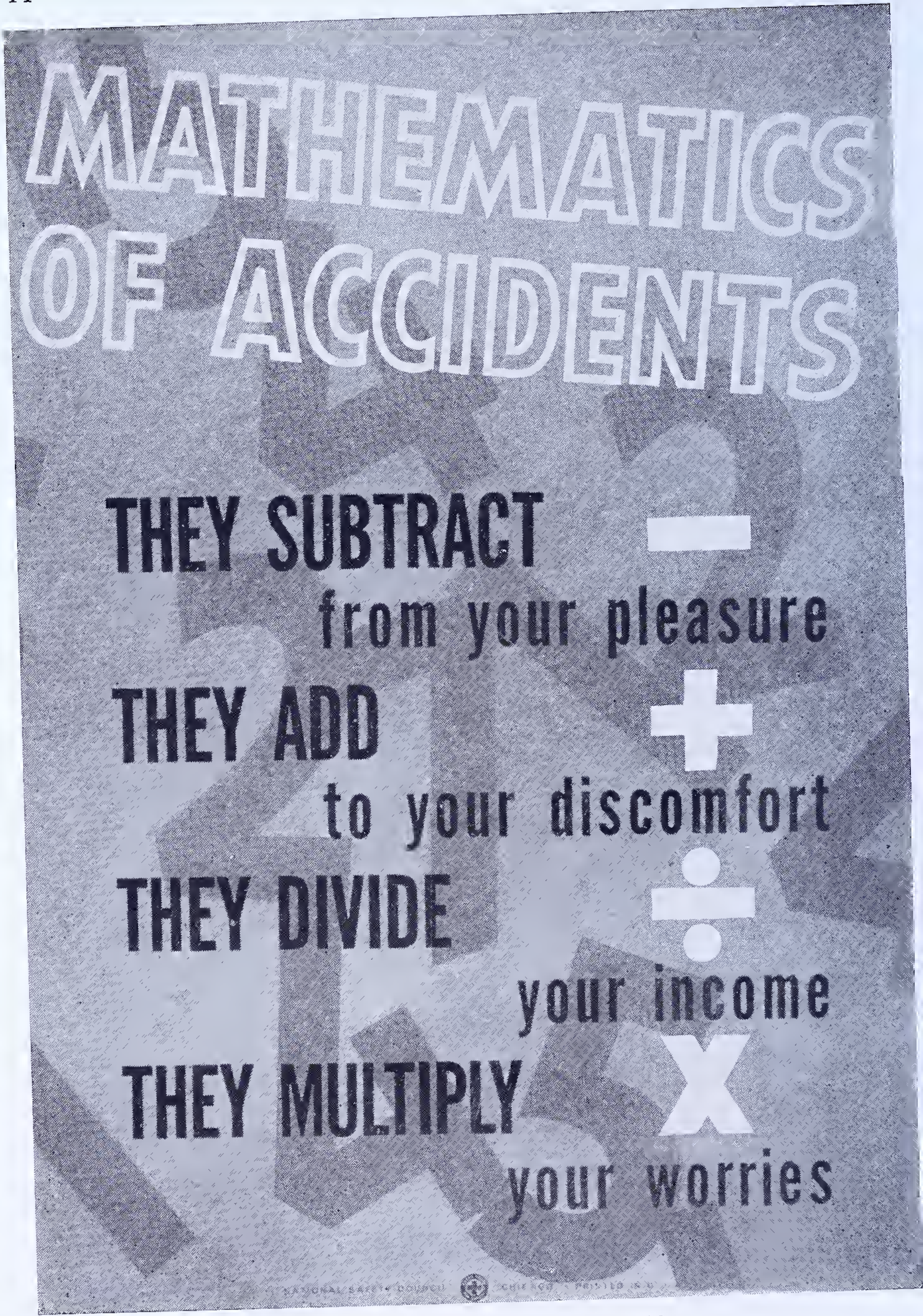
VII. Other Departments

The entire school plant may become the mathematics classroom. Many departments can furnish problem-solving materials for mathematical interpretation and solution. Each month, the director of the cafeteria can give the class that is studying logarithms problems to calculate for the report to the School Lunch and Nutrition Division of the Department of Public Instruction.

For example:

1. If the food for 4,560 students costs $1,232.65, how much will the food for 3,176 students cost?
2. If the labor for 4,560 students costs $574.76, how much does the labor for 3,176 students cost? The classes which have studied the slide rule check their results by that tool.

The machine shop is a never-ending source of problem-solving materials for which mathematical skills are essential. Here boys are trained



SYMBOLISM IN DAILY LIVING

to become machinists. Those who are also trained in drafting and mathematics frequently become designers of tools and machines. As an illustration, a tenth grade boy might bring a cylinder into the plane geometry class to find how geometry would help him locate the center of the shaft on which he was working in his shop class.

The Commercial Department can help some pupils operate computing machines. They might lend machines to the mathematics class for short periods so that the pupils could check their results when learning to use the slide rule.

Some boys from the physics classes asked their advanced algebra teacher to help them graph the equation for the sine curve. They wanted to get the connection between the equation $y = \sin x^o$ and alternating current. After they had put the curve on the graphing board, one boy drew a circle with radius equal to the same unit used in the graph. He marked 30°angles at the center; and at the intersections of the sides with the circle, he dropped perpendiculars to the X-axis. The next day the advanced algebra pupils continued their study and saw that the perpendiculars were also equal to the numbers used in their trigonometry tables the previous semester. Then they went down to the radio workshop to see alternating current produce a picture of the sine curve on the oscillograph screen. They listened to music from a radio station and watched its graph constantly changing as the amplitudes and frequencies varied. They also noted the graph made by the voice of Gabriel Heatter. They compared the graph of pure tones, electrically produced, with those of their own French horn, cornet, and Jew's harp. Finally, they saw all orchestral instruments combine in the interesting and beautiful patterns of Dvorak's Fifth Symphony. To such a group there will always be music in mathematics.

Coordination in an Entire School Faculty

Cooperative work by an entire school faculty around a central theme is readily feasible. A challenge for such coordination at an appropriate grade level is suggested by the following unit, "Family Life Education in the Total School Program." This has been produced by a State committee organized for this purpose. (See Acknowledgments, Appendix, page 294.)

Because of lack of space, sections relating to other subject areas are included only in the courses of study which are related to them. By common agreement, under capable leadership, department faculties may decide upon cooperative activity to help to build more firmly the family as the cornerstone of American life.

FAMILY LIFE EDUCATION IN THE TOTAL SCHOOL PROGRAM

Philosophy

What do we mean by Family Life Education?

Family Life Education is the total program within a school directed toward improving family living. It is the sum total of all concepts, attitudes, and skills gained in home and family living through a coordinated, well-planned program. Although courses in Family Relationships and Sex Education are sometimes called Family Life Education, the term Family Life Education is generally conceded to be broader in scope than any one course.

Who is responsible?

Everyone concerned with the growth and development of youth today will have a part in this program. To coordinate such a program in a school will mean the setting up of a working committee representing every subject matter area in addition to the school dietitian, nurse, guidance counselor, parents, and young people. The homemaking teacher because of her unique background will be a valuable member if not leader of this committee.

All courses offered in Homemaking,[1] i. e., the family's food, clothing for the family, child care and development, home management, home nursing, personal and social relations, will naturally be a fundamental and major part of this program in Family Life Education. To function constructively in the everyday living of boys and girls these courses must be family-centered. Music and art, too, have much to contribute to happy family living. Health and physical education will have a vital place in such a program. If the subject matter areas of English, science, mathematics, geography, modern foreign languages, social studies, are to meet the needs and concerns of youth today, they, too, must relate their discussions, readings, etc., to the family and the community of which the family is a part. Thus, every member of the school staff must recognize the part he or she has in developing improved attitudes toward the home. Every teacher has a contribution to make to family living by helping boys and girls understand the values of home life and how to become responsible family members.

Education for Family Life as far as the school is concerned should begin with the preschool child and continue through twelfth grade, for both boys and girls. The new *Elementary Course of Study,* Bulletin 233-B, treats the family life education under Chapter II, "Providing for Some Aspects of Education that Permeate the Whole Day," specifically, "Building Good Family Relationships through School Activities," p. 61. In order to meet more adequately the needs of the high school senior, a specific course should be offered in family relationships, giving emphasis to preparation for marriage. Many high schools are providing such courses under various titles, such as: Personal Living; Family Relationships; Senior Problems, Problems of Modern Living. This course can be taught by the teacher of homemaking or any other qualified member of the faculty. Such a person should be mature, well adjusted, skilled in human relations, willing to listen to young people.

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[1] Bulletin No. 325: *A Suggested Program in Homemaking for Secondary Schools,* Department of Public Instruction, Harrisburg.

EDUCATION FOR FAMILY LIFE

General Objective: **To strengthen and improve family living**

Specific Goals for Total School

To help boys and girls to

1. Understand themselves and how their behavior affects others
2. Recognize the needs and desires of others in the family group
3. Appreciate the value of good mental and physical health for personal happiness and wholesome family living
4. Understand the place and importance of children in the home
5. Gain an appreciation of the human and spiritual values in family living
6. Respect the contribution people of other cultures and races make to home and community living
7. Understand the need for cooperatively planned management in every home
8. Appreciate the need for beauty in the home
9. Recognize the responsibilities families have toward the community
10. Appreciate the responsibilities involved in parenthood and homemaking
11. Recognize factors which influence our standards for the selection of a life partner
12. Understand the home as a social institution and as a basic unit of democracy
13. Understand the value of the scientific approach to the solution of family problems
14. Develop ability to perform homemaking activities

SOME SUGGESTIONS FOR INTEGRATING FAMILY LIVING WITH THE SUBJECT MATTER AREAS

NOTE: In this report no attempt has been made to indicate the scope of subject matter in Family Life Education. Typical items have been selected to serve as illustrations.

INTEGRATION WITH MATHEMATICS

Specific Goals	*Content*	*Experiences*	*Teaching Aids*
To recognize the needs and desires of others in the family group To appreciate the responsibilities involved in parenthood and homemaking	1. Poor money management as possible source of much unhappiness in families Desirable planning of family members together for the spending of the family income. Such a plan a. Develops responsibility b. Provides for common needs c. Avoids selfishness d. Increases understanding of money problems 2. Learning to make wise choices as the key to satisfactions in family spending Analysis of items for which families spend money a. Food b. Clothing c. Shelter d. Operating expenses (1) Fuel (2) Gas (3) Replacing household equipment (4) Medical expenses (5) Education (6) Advancement (7) Recreation (8) Electricity (9) Telephone	1. Dramatizing an incident involving a problem of family spending as discussed in a family council 2. Discussing ways of distributing the family income among members of the family (allowance, etc.) 3. Emphasizing personal satisfactions and dissatisfactions involved in each method 4. Shopping for the family's food for a specified period Keeping a record of the amount spent 5. Panel discussion on owning vs. renting one's house 6. Computation of gas and electric bills 7. Computing interest on an amount deposited in a savings account in a bank and comparing with that invested in Government bonds	1. *Guiding Family Spending,* Miscellaneous Publication No. 661, U. S. Department of Agriculture 2. *The Buyer's Guide,* Consumer Education Study of the National Association of Secondary-School Principals, 1201 16th St. NW, Washington, D. C. 3. *How to Buy Life Insurance,* M. S. Stewart, Public Affairs Committee, 22 E. 38th St., N. Y. 16, N. Y.

INTEGRATION WITH MATHEMATICS—Continued

Specific Goals	*Content*	*Experiences*	*Teaching Aids*
To understand the need for cooperatively planned management in every home	3. Provision for savings as a part of family money management a. Reasons for savings b. Items for which families save c. Methods of saving (1) Bank (2) Insurance—a long-range plan (3) Government bonds 4. Wise money management includes a *plan* for spending a. The family budget—a *guide* for spending b. The same plan will not satisfy any two families 5. Median income in the U. S. is $3,000.00, which means that half the incomes are below $3,000.00 Forty per cent of families have a net worth of $5,000.00; ten per cent over $25,000.00 6. Credit as a family problem Reliable sources for borrowing money vs. "loan sharks" a. Dangers involved in installment buying b. One-third of families spent more than they earned in 1949	8. Discussing various types of insurance which families carry 9. Examining personal account books 10. Examining household account books 11. Discussing advantages of keeping a record of expenditures 12. Keeping a record of your personal expenditures for a week; then for a month. Studying these accounts to see how you could have spent more wisely 13. Planning a budget for a "case" family. Considering income, number and ages of family members, and the locality 14. Computing the cost of an article purchased on a cash basis with the same article bought on an installment plan 15. Computing interest charged on a loan made through a bank. Comparing this with interest charges of a loan company 16. Finding out government regulations on installment buying	4. *Buying Insurance*, Consumer Education Study of N.A.S.S.P. 5. *Using Consumer Credit*, Consumer Education Study of N.A.S.S.P. 6. *Small Loan Laws of the U. S.*, Pollock Foundation for Economic Research, Jaffrey, N. H. 7. *Credit for Consumers*, L. R. Foster, Public Affairs Committee, 22 E. 38th St., N. Y. 16, N. Y. 8. Have representative from a reputable credit association or from the credit office of a department store discuss credit buying and credit rating as these affect families

ENRICHMENT[1]

Are we doing right by the top 20 per cent of our students? Nicholas Murray Butler once spoke of America as the "best half-educated country in the world." Generally speaking, we have geared our teaching to the average or to the slow pupil. The gifted child is left to himself to use a minimum of effort to achieve a maximum result.

The purpose here is to show how it is possible to motivate or to challenge capable pupils to do projects or research problems different from the regular course of study. It should be clearly understood by all teachers who try these approaches that the sole purpose is for a better understanding and appreciation of the unit of work being studied and is not just for "busy work." While some of the suggestions may seem to be recreational, their aims are not recreational for the student but are for his own understanding of the power and the beauty in mathematics.

Leadership Activities

Teachers facing the prospect of improved individualization of instruction are often overwhelmed by the thought of trying to direct thirty different lessons simultaneously. This becomes less difficult as the teacher ceases to dominate the class and considers his role more as that of a guide. The presence of a number of gifted pupils in his algebra class affords an opportunity to make this transition.

Once the basic explanation has been made and the class is engaged in supervised study, the teacher can work with a group of superior pupils to give them some guidance in working out leadership activities such as those suggested below:

1. Individuals may present to the class some related aspects of equations. For instance, from their research they should identify and tell the others about the need for equations.
2. As "captain" of a row or a group, a gifted pupil may help others during the supervised study period. The teacher first checks with these captains to make sure that they have the proper understanding and then gives them the responsibility of helping their designated groups.
3. While most people can determine the correctness of the solution of an equation by using an answer book or by checking, only those with real understanding can tell why a particular solution is wrong. The gifted may be given copies of equations worked out incorrectly (preferably taken from errors made by members of the class) and then requested to write an analysis of the errors.

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[1] From *Programs for the Gifted*, Educational Service Bureau, University of Pennsylvania, 1950.

4. Encourage the gifted to make out suitable equations to be used the following day as a review or as a checkup. Further leadership experience can be gained by allowing different individuals to take over the administration of these review exercises.

The teacher should be careful not to look upon these activities as mere busy work or as devices to relieve her of basic teaching responsibilities. The sample activities presented here offer opportunities for experiences in creative work, in developing critical analysis, in fostering understanding, and in leadership training. The teacher needs to keep these possibilities in mind as she develops, supervises, and follows up the individual work.

Research Activities

Research work seems particularly adapted to this group of gifted pupils, since they generally have an untiring desire for knowledge and an outstanding degree of originality, resourcefulness, initiative, and imagination. This should produce a feeling of security and self-reliance, encourage creative talents and special abilities, and increase leadership qualifications.

The projects should be only suggestions, with the details of organization and presentation left to the ingenuity of the student. Upon completion, each student should have an opportunity to present his problem and his solution to the group for critical and complimentary comment.

Possible Problems in Ninth Grade Algebraic Equations

1. *Algebra is a form of shorthand.* The student can take an equation, write its equivalent in sentence form in English, then in several other languages, and possibly in stenographic shorthand. If he has had the languages, he should make his own translation; otherwise it will be necessary to enlist the aid of others. This activity can be presented by means of posters, by cards with flag of the country and an accompanying statement of the equation in the language, by maps with the statement written across the country, or on the blackboard—one each day—the class to identify the language in which the equation is written. The problem can be enlarged by presenting the statement with the name of a mathematician of the country speaking that language, or by naming a contribution of that country to mathematics. The pupil doubtless will think of many other possibilities.

2. *Symbolism plays a large part in algebra.* Tracing the development of symbols with dates, countries, famous names, etc., will give ample chance for research. For the student who excels in art, this would make an interesting frieze or series of panels for a mathematics room. Illustrations of the same equation written with symbols in use in various periods

could be used. This material can also be written in a form similar to the historical comic.

3. *Words are the keys to many interesting pathways.* Even the words in algebra can lead to fascinating bypaths. Start the pupil tracing the origin of a few common words in connection with equations, their various ramifications and meanings before reaching the present form. The words can be presented as a simple dictionary, or as a picture dictionary.

4. *The history of the equation* and its various types might include its transfer from country to country.

5. *For the bulletin board,* to stimulate other gifted pupils, or for mathematics clubs, mathematical axioms can be written as cryptograms, lists of mathematical terms can be deciphered from jumbled letters, names of mathematicians can be guessed from original clues written up by individuals, and a list of countries and mathematicians or of mathematicians and their contributions can be matched.

6. Finally, where it is possible to cover only the minimum course of study, *there is much valuable enrichment in the course itself.* The gifted pupil can go into the more difficult types of equations which were not included in the limited course without infringing on his second course in algebra.

Homemade Word Problems

Enrichment can never successfully substitute for careful meaningful development, for testing, for class organization, or for teacher personality. But enrichment can give deeper, wider connections with mathematics. One means of enrichment deals with solving of verbal, or reading, or analysis problems.

In reading problems the processes of sound reasoning and clear thinking are easily exhibited. Ideas of dependence, correspondence, changes, and relationships are encountered. The stronger pupil who has learned these methods of functional thinking could therefore be trained to apply the techniques to social and economic problems encountered by adults. Such problems offer the opportunity for training in methods of deductive reasoning with verification of each step in the solution of an equation.

Problems are of greater interest to children if the meanings of words are studied, as needed, and if the problems are related to pupils' interests, hobbies, and talents. *Let the children make up problems and work each other's problems.* They will develop a mathematical frame of mind, a method of doing things thoroughly and accurately. Since a great part of elementary training in algebra is concerned with learning how to translate English to algebra and back again, this assignment of homemade verbal problems makes the mathematics teacher responsible, to this extent, for the training of the student in English.

Making up word problems helps the better student appreciate the equation as the most powerful instrument we have with which to solve problems. As most texts point out, the equation is compared to a balance with equal weights on the two sides; but, as Osborne puts it, it could be thought of as a mighty press or vise. Put the right facts into it, and it will crack the most difficult problem.

Following are several illustrations of word problems made up by a few of the students in one Algebra I class:

1. In a piece of music the first measure has a certain number of eighth notes and a different number of sixteenth notes. The second measure has three less eighth notes and four times as many sixteenth notes. How many are there of each in the first measure if there are three beats to a measure and each quarter note gets one beat?
2. One morning the owner of a swimming pool finds it clogged up. After cleaning it out, he opens the two drainage pipes, knowing that they can empty it in 12 minutes. However, after the pipes have been open for 3 minutes, one pipe suddenly is stopped so that the emptying takes 18 minutes in all. How long would it take to empty the tank if only one pipe were opened at the start?
3. One man can cut and trim an entire golf course in eight days working alone. After he had worked alone for four days, a helper joins him and they complete the job in three days. How long would it take the helper working alone to do the job?
4. The owner of a baseball team is confident that if his team has a .500 average, it will have a good chance to be in the first division. If the team has won 40 games and lost 50, how many games does it have to win in succession to reach a .500 average?

Correlation with Other Subjects

One method of enrichment for the gifted child is to correlate mathematics with other school subjects. The individual is encouraged to do research work which involves the particular mathematics course he is currently pursuing and some other course. In doing so, the student might be given the opportunity in his mathematics course, which is highly objective, to evaluate some superstition or subject idea. He will thus experience thorough self-satisfaction coupled with a broad intellectual and aesthetic appreciation of mathematics.

Much is to be gained in additional knowledge and appreciation, but the extrinsic values very likely outweigh the more formal and routine factual work. Learning research techniques and gaining the ability to do work of high quality are especially important to the gifted pupil.

The teacher must avoid helping too much; he suggests and guides while the pupil reaps the full measure of satisfaction derived from having planned and carried out his own learning activity.

For example, after conferring with a gifted pupil, we might find that he is particularly interested in ancient history. We could suggest that he might gather data from books dealing with the history of mathematics. We might also add that for a satisfactory project he could give an oral or a written report, make some mechanical device, make a poster, or teach the class a new idea or concept.

PROVISION FOR THE BELOW-AVERAGE STUDENT

The problem of educating the slow learner in the secondary school arose when high school education became available to all youth instead of to the select few. Today in the secondary school there are large groups of pupils who have low intellectual ability, who show little interest, whose aptitude or preparation is meager.

These pupils are not able to do successfully the mathematics of the traditional high school courses because such courses are geared to college preparatory or technical courses. The problem is how to plan mathematical situations that will give the slow learner functional experiences on his level of ability and interest so that he can succeed in developing mathematical competence of value in practical situations. Many of these youth become our most successful citizens and employ skilled mathematicians to do their work for them.

Administrative Approaches

1. The slow learner takes the same subject with the entire group but the assignments are varied in an attempt to reach his individual functional needs. This system tends to be ineffective because of the great amount of time needed for individual diagnostic and remedial work.
2. The slow learners are placed in special classes for definite remedial and individual work. The program is of a coaching type to try to rebuild the pupil's understanding so that he may rejoin the regular group.
3. The recognition of a division of mathematics with suitable courses, content, and methods of instruction adapted for those with I.Q. from 80 to 110 who do not readily learn formal abstract mathematics.
4. The group should not have more than 22 pupils.
5. A testing and case study method should be used to obtain an understanding of the individual pupil. The main factors to be studied are mental age, personality reactions, home background, health, school records, subject grade levels, prognostic tests, etc.

Trends and Viewpoints

1. To preserve good mental health and individual morale, pupils must experience success more often than failure.
2. The child can be taught only at his instructional level.
3. Individual aims and functional objectives for each child must be set up in the instructional program.
4. Appropriate books and work sheets must be provided at pupils' instructional level.
5. There must be individual pupil-teacher conferences.
6. Record should be kept of pupils' individual progress.
7. Work should be as interesting, practical, and lifelike as possible.
8. Pupils should be praised and encouraged as much as possible when they deserve it.
9. An approach from many points of view is necessary. Discussions, activities, investigations, experiments, reading, computation, visual material, interviews, and field trips should be used to make situations meaningful. Teaching must be related to life situations that the pupil can recognize. There should be emphasis on practical, concrete devices for teaching.
10. Instruction cannot be hurried. Time and patience are necessary for the slow-learning group. Thoroughness rather than speed is important.
11. Reading material in nontechnical vocabulary is needed. As yet too little material of this type has been developed.
12. Meaningful repetition and review are needed whenever specific work indicates this procedure is advisable; it should be on an individual basis. Review should be connected with the activity of the unit and not with an isolated situation.

Psychological Viewpoints

1. Work must be child-centered not teacher-dominated. The pupil must participate and perform. The teacher's job is to motivate, organize, and direct.
2. The principles of learning define the teaching process:
   a. Readiness
   b. Participation
   c. Interest
   d. Value and satisfaction derived from realistic experiences
3. The work must have significance to the learner.
4. Activities should be practical life situations.

5. The student must be taught as an individual.
6. The unit, the problem, and the project methods are most favorable teaching situations.
7. The unit method:
    a. Planning—Teacher and pupils cooperate
    b. Motivation—Selection and introduction of unit theme
        Situations that have social value are effective
        Objectives—Selection
        Materials—Selection
    c. Work stage
    d. Presentation of ideas—Utilization of many aids and devices
    e. Evaluation—The pupil should participate in the evaluation of his educational experiences

Content Determination Must Be Based on Mental Age, Interest Level, Present Usefulness to Learner, Future Use to Learner

In content the following areas are needed:

Concepts needed for fundamental operations in addition, subtraction, multiplication, division
Use of simple fractions and decimals
Use of system of measuring and weighing materials
Making change
Telling time and using the thermometer
Reading gas and electric meters
Using simple percentage
Reading and making picture, line, and bar graphs
Reading and understanding large numbers
Use of numbers in recreational situations
Drawing to scale
Use of money
Home and business arithmetic situations
Simple formulas and equations
Simple applied geometry concepts
Insurance, banking, taxation

SOME DO'S AND DON'TS

In a program of meaningful mathematics, a basic task of the teacher is to analyze the instructional problems involved in any unit or topic in order that he may anticipate the particular difficulties and errors that are likely to occur in connection with it and then teach the unit in such manner that as far as possible the difficulties and errors may be avoided.

It is the purpose of this section to point out and discuss briefly some of the many difficulties, potential errors, probable pitfalls, and controversial procedures and techniques that will be met in teaching secondary mathematics.

Transposition

In early work in the study and solution of equations, emphasis is placed upon "keeping the balance undisturbed," or "preserving the equality." This can be done mathematically by the use of axioms or by what amounts to the same thing, the application of the processes inverse to those which bind numbers or letters to the unknown x.

Example:

Start with	$3x + 4 = 10$	
Subtract 4	$-4 = -4$	.......Subtract 4 from each member *(The inverse of addition is subtraction.)*
	$3x = 6$	.......*If equals are subtracted from equals the remainders are equal.*
Divide by 3	$\frac{3x}{3} = \frac{6}{3}$	.......Divide each member by 3 *(Inverse of multiplication is division.)*
	$x = 2$	.......*If equals are divided by equals, the quotients are equal.*

However, some teachers very soon or even in the beginning announce the "magical short cut," transposition, and from then on students solve equations with little or no mathematical thinking or understanding. They "get rid of" an unwanted number or letter on one side of the equation by mechanically following the directive "Move it over to the other side and change its sign."

Although the principle of transposition can be established by a logical proof, it leads to errors because of the mechanical way in which it is used by many students. Even in such simple equations as

$$3x = 5 \text{ or } \frac{x}{3} = 5,$$

some one is almost sure to say "transpose the 3." We may profitably eliminate the use of the word "transposition" from high school mathematics, and teach equations and formulas by a method which will develop understanding and mathematical thinking, namely, the use of the axioms of addition, subtraction, multiplication, and division. "Fortunate is the high school student who has never heard the word *transpose*."[1]

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[1] M. H. Ahrendt, "Pet Peeves of a Mathematics Teacher," *Mathematics Teacher*, Vol. 39, (February, 1946), pp. 71-74.

Cross-Multiply

A proportion is usually written as an equality of two fractions or ratios. In a proportion the product of the means equals the product of the extremes. In clearing this equation of the fractions some students say "cross-multiply." This is not a mathematical expression and consequently lends itself to many misinterpretations.

We find the term "cross-product" used in mathematics texts in connection with the product of two binomials, but this is not what students and teachers mean when they say "cross-multiply."

The real danger lies in the fact that very often students who use this expression will attempt to apply the principle incorrectly to fractional equations and other expressions which are not in the form of a proportion, such as the following:

Student might mistakenly write:	*Incorrectly as:*
$\frac{2x}{3} = \frac{x}{2} + 4$	$4x = 3x + 4$
$\frac{6}{x+2} + \frac{8}{x+1} = \frac{24}{x+3}$	$48\ (x+3) = 24\ (x+2)\ (x+1)$
$\frac{3x}{2} \times \frac{9}{8} = \underline{\qquad\quad}$	$24x = 18$

Teachers of mathematics should emphatically discourage the use of any nonmathematical terms and procedures such as "cross-multiply."

Cancellation or Cancel

The word "cancel" to many students means nothing more than crossing out numbers that look alike, regardless of mathematical laws. "Cancellation" is a word that has long been used in mathematics to mean the process of reduction of fractions before multiplying. The presentation of the so-called short cut before the underlying principle is clearly understood and before there has been sufficient practice in applying the principles, results in many errors and absurdities such as the following:

$$\frac{3}{\cancel{x}} + \frac{\cancel{x}}{7} = \frac{3}{7}$$

$$\frac{\cancel{a} - b}{\cancel{a} - c} = \frac{b}{c}$$

$$\frac{\cancel{8} + 5x}{\cancel{8}} = 5x$$

$$\frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{6}} \times \frac{\cancel{2}}{5} = 5$$

$$\frac{\cancel{a}}{\cancel{b}} \times \frac{\cancel{b}}{\cancel{c}} \times \frac{\cancel{c}}{\cancel{a}} = 0$$

"Cancel" is also used by many students to indicate that the sum of two or more numbers is zero and they are crossed out in the following manner:

$$\cancel{6x} + 8y - \cancel{6x} + 4 = 8y + 4$$

The use of the same word "cancel" for two different processes or concepts results in confusion for many learners and it should not be used until the underlying principles have been thoroughly developed, if at all. The teacher should make certain that the student understands the principle: *Dividing the numerator and denominator of a fraction by the same number (other than zero) does not change the value of the fraction.* In solving a problem such as the one shown below, the confusion is generally cleared up if instead of saying "cancel" the student says "divide the numerator and denominator by 5." The quotient 1 should be indicated especially in the beginning stage. $\frac{3}{\cancel{5}} \times \frac{\cancel{5}}{4} = \frac{3}{4}$

"Cancellitus" is a mathematical disease that tends to cause its victims to fall into the pit of misunderstanding.

Order

$4 + 6 \times 2 = 16$ (not 20) $8 - 6 \div 2 = 5$ (not 1)

That is, *(by agreement among mathematicians) multiplication and division should be performed before addition or subtraction when several operations are indicated, as above.*

$\sqrt{a^2 + b^2}$ does not equal $a + b$; nor does $\sqrt{9 + 16}$ equal $3 + 4$, or 7.

What is the square root of 36? Since $(+6) \times (+6) = +36$, and $(-6) \times (-6) = +36$, the $\sqrt{36}$ is either $(+6)$ or (-6) When the radical sign has no sign before it, the positive square root only is understood.

Thus, $\sqrt{49} = +7$, $-\sqrt{49} = -7$ and $\pm\sqrt{49} = \pm 7$

The positive square root is called the *principal root.* Hence $\sqrt{49}$ refers to the *principal square root* $(+7)$ only.

Graphs

1. The scale should be selected carefully so that the graph covers most of the sheet.
2. "The graph should have a title which should be complete and clearly stated, so that misinterpretations would be impossible."[1]
3. "In picturing areas as volumes to represent quantities, care should be exercised to prevent giving false impressions. This applies especially to pictograph area and volume pictures."[1]

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[1] Edwin Eagle, "Toward Better Graphs," *Mathematics Teacher,* Vol. 35, (March, 1942) pp. 127-131.

False impressions can usually be avoided by using pictures of the same size, each picture to represent a given quantity. The number of pictures used is determined by the total amount to be represented.

Signed Numbers

Encourage use of the words *positive* and *negative* when referring to numbers and *plus* and *minus* when referring to an operation, as for example:

$+5$ is greater than -4, positive 5 is greater than negative 4.
$8-5=3$, 8 minus 5 equals 3.

Formulas

Emphasize the fact that a formula is an equation. In solving for a particular unknown (in terms of the other quantities) the same procedure is used. It is usually desirable to solve for the required unknown before substituting given values of the other quantities, especially when the evaluation is performed by logarithms.

Vocabulary

Increased emphasis should be given to the understanding and use of a good mathematical vocabulary, and the elimination of absurd and meaningless words and expressions, as for example:

1. "Invert fraction" or "write reciprocal" is surely better than "turn it upside down."
2. When reading a product such as $x(a-b)$, do not say "x onto $(a-b)$," "x *into* $(a-b)$," "a parenthesis $a-b$," etc., but these are not uncommon.
   It should be read, "x times the quantity $a-b$."
3. In reading the fraction $\frac{a}{b}$, "a divided by b" is preferable to "a over b."
4. In reading cos x, tan x, etc., say "cosine x," "tangent x," not "cos x," "tan x."
5. In finding areas and volume, we still hear absurd statements like: "Feet times feet equal square feet." "Feet times feet times feet equal cubic feet." It is the task of the teacher to reteach if necessary the principles involved in the use of the fundamental processes involving concrete and abstract numbers. The pupil must understand that *a concrete number can not be multiplied by a concrete number.*

MAKING MATHEMATICS MEANINGFUL

To make mathematics more meaningful, a program should first find out what pupils need to know in order to do better what they are now doing and what they will do later as homemakers, workers, and citizens. Second, it should

recommend the best ways to teach the most functional mathematics to them. The program must represent an integration of: (1) pupil needs, (2) social values, (3) skill in using mathematics, (4) functional learning activities, and (5) means of comprehensive evaluation.

The problems involved in planning require the best that modern research and grass-roots cooperation can provide. This problem is uncovered in the foregoing discussions of "The Needs of Youth," "Purposes of Mathematics Education," "Articulation," "Provision for the Below-Average Student," and "Some Do's and Don'ts." Because of their close relationships, these problems may be considered together.

While secondary schools have always achieved results of significance, there is room for improvement. Our society has been undergoing such rapid change that it is difficult for schools and mathematics departments to adapt their programs fast enough to provide the "education for doing" that is needed. In addition, educational research has been discovering much about what learning is and how it can best be created.

Research workers in education and teachers in action, individually and in groups, have evolved several basic points of view which, for them, create more effective learning. These results of a half century of progress are generally accepted and recorded: (1) in modern textbooks on educational psychology,[1] (2) in books on teaching methods,[2] (3) in the 1950 edition of the *Evaluative Criteria* of the Cooperative Study of Secondary School Standards,[3] and (4) in reports of state and national committees.[4] These sources present points of view on which each teacher for himself must make decisions.

Are the prestige and consensus of these sources significant? Do they represent the development that comes from scientific research and growth in any profession? Do these sources indicate the "unity out of diversity" which people need to go forward together? Has there been evolved "a scientifically derived philosophy of education"? Is a gradual achievement of these expressed points of view possible?

Many mathematics teachers are adapting their work to these points of view. Others are dissatisfied with present results and are looking for more valid curriculum practices. Curriculum improvement programs have been organized in nearly every state and city so that the behaviors of good citizenship and scholarship may be better developed and so that public funds for education may be spent more effectively. In Pennsyl-

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[1] Arthur I. Gates, and others, *Educational Psychology.* New York, The Macmillan Company, 1948.

[2] *Learning and Instruction.* Forty-ninth Yearbook of the National Society for the Study of Education. Chicago, Ill., University of Chicago Press, 1950.

[3] *Evaluative Criteria.* Washington, D. C., Cooperative Study of Secondary School Standards, 1950. (See Chapter IV, "Evaluation.")

[4] See yearbooks of the National Society for the Study of Education.

vania, in addition to the State Curriculum Improvement Program, four school study councils and twelve annual principals' workships have been organized to stimulate the changes which are represented. Hundreds of local faculty programs are focused toward the development of programs of modern education.

Some of these points of view are listed below in order that the user of this bulletin may consider them and may understand some of the bases for suggestions which are made in programs of curriculum improvement and in the pages which follow.

Objectives in Terms of Growth in What the Learner Does

Learning is a change of behavior—thinking, feeling, and acting—which is produced by what the learner does.[1] There is a difference between teaching for knowing and teaching for doing. The dual problem of helping youth to mature and to acquire desirable behaviors, including skills in the *use* of mathematics, is broader than has long been assumed.

Teaching for doing involves the principles that:

1. Objectives are best expressed as desirable changes in what the learner actually does.
2. Desirable changes in what the learner does do not develop automatically as by-products of the teaching of subject matter.
3. Each type of behavior, including skills in the use of mathematics, requires direct provision for its practice in active learning situations and in types of evaluation.
4. Essential values in education lie in teaching the understanding of functional subject matter skills and concepts and providing practice in the application of them to everyday life in school and in the learner's environment.

The teaching and the testing of subject matter data alone have been found to have little effect on functional learning. Teaching that does not change what the learner does—thinks, feels, and acts—does not achieve the purposes of either life adjustment or precollege education. Genuine understanding does not extend far beyond the limits of the learner's actual experience.

The following considerations are derived from this fundamental agreement:

1. *Functional Understanding*

There is a distinction between the memorized learning of inert ideas and the functional behaviors and transfer values which are developed

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[1] *Learning and Instruction,* see footnote, page 31.

A FIELD TRIP—ARTICULATION WITH USE

through meaningful learning activities in unit study. Learning-by-doing activities which are meaningful to the learner in terms of his life and needs create interested participation and real learning. The learner adjusts better to his environment. The satisfaction of immediate needs provides gradients for further achievement in the use of mathematics and for greater goals.[1] The pupil learns and remembers what he uses and needs to know. How much learning will remain after the pupil leaves school should be a matter of great concern and of frequent evaluation.

2. *Meaningful Learning Activities*

Learning situations are meaningful to the learner insofar as they are—for him—lifelike and worth while. He may instinctively resist learning that seems to him useless and trivial, which lacks unity and relation to his own environment or ambitions. A meaningful unit, problem, or project challenges the will to learn, more than do awards or coercion. Discipline is best when it is inherent in the meaningfulness of the learning.

3. *Learning Activities*

Without the participation of the learner in many types of learning activities, there is excessive memorized learning. Daily assign-study-recite-quiz procedures encourage temporary verbal mastery. Desirable behaviors, including skills in *using* mathematics, are developed by practicing them. Learning for doing involves much doing in the learning.

4. *Problem-Solving*

Learning activities of a problem-solving unit type are especially productive of the critical thinking and the functional use of mathematics needed both in school and in life. Problem-solving behaviors are developed through learning activities in which they are practiced.

5. *Planning*

Teacher preplanning and pupil-teacher planning are both essential for well-motivated class activities.[2] Pupil participation in planning develops pupil concerns and efforts beyond the shallow ones of getting a mark to please the teacher and to add more credits. The pupil must share if he is to care as much as he should. In educating for citizenship, his whole development as an individual is a matter of concern. He needs practice in planning and in evaluating.

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[1] *Educational Psychology*, see footnote, page 31.
[2] *Evaluative Criteria*, see footnote, page 31.

6. *Successful Teaching*

   Learning has been defined as the behavior changes—thinking, feeling, and acting—which result from what the learner does. Successful teaching, then, involves setting the stage with problems, learning activities, and subject matter that will assure pupil reaction, practice, and attainment of the desired behavior changes, including skill in *using* mathematics. Classrooms should be transformed from lesson-hearing rooms into learning laboratories.

The above statements cover much research and careful thinking by many persons. Coming as they do from well-recognized sources, they may be helpful in solving many problems of curriculum improvement. Teaching is not always successful in terms of pupil mastery of desirable behaviors. Suggestions from such authoritative sources as those cited may point the way to better results through more meaningful and varied pupil learning activities. The suggestions become more clear as they are illustrated in practice. To provide ideas for the gradual curriculum improvement which is necessary to carry them out and to furnish examples of some of them in operation are the purposes of this cooperative publication.

Experience has shown that the study of principles does not by itself result in curriculum improvement. It is best to start with one or more specific individual or group problems that present challenge and merit action. However, attacking problems without reference to basic principles does not often result in improvement. A two-way procedure, from problems to principles and back again, may be most effective.

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CHAPTER II

IMPROVEMENT IN TEACHING

DISCOVERING FOR ONE'S SELF IN THE CLASSROOM

In the tryout and evaluation of new materials and methods there are two general types of self-discovery in the classroom. *First, there is an over-all, direct type.* When a school faculty, a faculty committee, or an individual teacher decides upon objectives to be achieved, materials for a direct attack upon the objectives are prepared. These materials usually take the form of (1) improved lesson plans or (2) units for which the objectives, or phases of them, are titles and bases for planning. These materials and methods must then be proved by teachers' tryout and evaluation in the classroom.

***Second, there is the kind of classroom tryout and discovery which is of an incidental type.* Problems considered are those that arise out of the local situation and seem of importance to teachers. This problem-solving is constantly under way. As a type of informal "action research." it is of great value. Its steps may be described by such simple words as: (1) an idea that something can be done better, (2) a tryout of what seems better, (3) some data are gathered, (4) judgment is used to measure improvement, (5) more know-how and professional satisfaction result. This "action research" can be done by every teacher with the approval or within the policies of the school administration. Without it, curriculum improvement is not possible.[1]**

Secondary school teachers and principals must be free to try out better methods and materials without threats to their security. Otherwise the individual initiative and creativeness will be a lost virtue. Administrative encouragement and provision of time and materials promote faculty and individual self-discovery on problems of concern to teachers, problems for which they request help. What are these problems? How can they be solved?

Problems of Teaching

We want to know how to deviate from the textbook without getting lost; how to introduce and teach a unit; how to plan with pupils . . .

We want to know how to change the morale and behavior of groups; how to use the principles of group dynamics; how to help groups reach decisions and evaluate their own work; how to relate ongoing activity to the problems, concerns, and tensions of pupils; how to work with a small group in a classroom and, at the same time, keep other pupils profitably busy . . .

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[1] Stephen M. Corey, "Action Research, Fundamental Research, and Educational Practices," *Teachers College Record,* Volume 50, May 1949.

We want to know how to spot and use community resources; how to find people who know the answers to our problems and how to get their help; how to build units on problems not found in textbooks; how to provide school experiences that will help pupils toward maturity.

These were the grouped replies of three hundred classroom teachers to an inquiry on what help they wanted most.[1]

Similar "how to" problems faced by many teachers today have been created by the impact of modern functional education upon an inherited educational process. Statistics which reveal startling personal and social shortages have made their solution a matter of deep concern. Their consideration by school faculties and individual teachers is presented as a challenge in this bulletin. For their solution, experimentation must move into the classroom.

For a long time, the research pattern used in education has been based upon the methods which have developed progress in the physical sciences. Controlled statistical experimentation has established the validity of many educational principles. However, these reports are often filled with statistical jargon. The research value is generally incomplete because the classroom application is seldom accomplished. Because of this, the knowledge of what education should and can do has far outrun its practices.

This bulletin and other reports in the literature of education indicate how some individuals have solved the "how to" problems which teachers present. The results, as recorded, are created by factors of the situation which produced them—the teacher, the pupils, the school, and the equipment. However, they provide stimulation and suggestions for every teacher. *Bulletins and reports are of significance not in what they do for people, but in what they get people to do for themselves.* Each teacher should, through experimentation, try out and reconstruct the experiences of others and develop many of his own in his classroom, if "how to" problems are to be solved.[2]

Even though first creative efforts may not achieve full success, patience may be needed. As in any kind of problem-solving, learning-by-doing—particularly learning-by-trying—is the best approach. Few experiments in education end in failure. There is much room for improvement. The zeal, vigor, and enthusiasm which the experimenter himself contributes result in better total development of students no matter what results may be measured.

Some direction for self-discovery by a teacher may be indicated by the following chart:

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[1] Vernon L. Replogle, "What Help Do Teachers Want?" *Educational Leadership,* Volume VII, No. 7, April 1950.

[2] A. W. Foshay and James A. Hall, "Experimentation Moves into the Classroom." *Teachers College Record,* Volume 51, No. 6, March 1950.

A TEACHER'S SELF-RATING CHART

	CHECK		
	Yes	*?*	*No*
1. Can I define the actual pupil behaviors—thinking, feeling, and acting—which I am trying to develop?	______	______	______
2. Do I guide learning activities in which desirable behaviors are practiced?	______	______	______
3. Can I depart from daily recitation (textbook assign-study-recite-quiz procedure) without getting lost?	______	______	______
4. Can I plan and guide an experience SUBJECT MATTER unit?	______	______	______
5. Can I plan with pupils and guide an experience LIFE ADJUSTMENT unit?	______	______	______
6. Do I know how to use the principles of group dynamics?	______	______	______
7. Do my pupils feel free to discuss their problems with me?	______	______	______
8. Can I relate content teaching to the adolescent problems, concerns, and tensions of pupils?	______	______	______
9. Can I subgroup, form committees, etc., and keep all working on a well-motivated level?	______	______	______
10. Do I know how to spot and use community resources?	______	______	______
11. Do my students apply the principles of my subject to their own problems?	______	______	______
12. Do I involve the introverted and "isolates" in group activities?	______	______	______
13. Do I teach my pupils how to read and study my subject?	______	______	______
14. Do my students use good English in oral and written reports?	______	______	______
15. Do I continually invite student problems for discussion and problem solving?	______	______	______
16. Are my students having actual, frequent practice in critical, inductive reasoning?	______	______	______
17. Do I work with a club to further special interests?	______	______	______
18. Have I had adequate preparation in fields which are related to my subject?	______	______	______
19. Do I have an adequate knowledge of my object—the learner?	______	______	______
20. Have I had intensive and modern preparation in the subject I teach?	______	______	______
21. Do I relate my work to that of other subject teachers in my school and request similar cooperation?	______	______	______
22. Do my students understand the vocational opportunities in my subject field or work to which it may lead?	______	______	______
23. Do I praise more often than I blame?	______	______	______
24. Do I subgroup to meet the special needs of vocational and precollege students?	______	______	______
25. Do I make a special effort to locate and encourage talented youth?	______	______	______

GROUPING OF STUDENTS IN THE MATHEMATICS LABORATORY

Grass-roots faculty tryout and discovery are essential parts of a forward-looking program that seeks to achieve educational objectives and the more effective use of public funds. Many experimental programs stem from public dissatisfaction and public help.

It is significant that parents express the criterion of "ability to work with pupils" as the highest measure for teacher selection.[1] Much of this ability comes from the knowledge of and provision for the needs of youth. This is the concern of the following sections:

Improving Study

A Laboratory Approach to Mathematics

Development of Units

Individualizing Instruction

Using Community Resources

Providing Guidance

Factors in How to Study

When teachers gave "poor work habits" and "inability to study" as the cause for pupil failure in school work, the Canadian Council for Educational Research analyzed 38 books devoted to teaching how to study. A total of 517 suggested study habits and skills were found to be mentioned. In order of the frequency of appearance, here are the study suggestions mentioned 20 or more times, as reported in the *Educational Review,* Saint John, New Brunswick, Volume LIX, Number 4, page 11.

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[1] "What the U. S. Thinks of Its Schools," a Roper Survey. *Life,* October 16, 1950.

Study Technique	*Frequency*
Seek and acquire a motive	34
Have a neat and carefully thought-out system of taking notes	29
Plan a reasonable time budget for day and week	29
Use ways of developing concentration	27
Take positive attitude toward task	26
Realize necessity of attitudes of alertness, confidence, attention in reading	26
Exercise physical care during study	25
Improve vocabulary	25
Have proper place to work	24
Avoid distractions from within	24
Read critically	24
Organize material read under headings and topics	24
Apply knowledge learned	23
Avoid immature habits of reading	23
Use differential rates of reading for different types of materials	23
Make use of tables of contents	23
Organize the preliminary notes; use outlines	23
Do preliminary bibliographical work for written reports	23
Select main points of a lecture and take adequate but not too copious notes	23
Use internal motives	22
Use objective study scales, questionnaires on study habits, spelling graphs	22
Use various types of speed exercises, drills, tests in reading	22
Read for different purposes	22
Mark books intelligently	22
Know the necessary preliminary activities with a new book	22
Collect ideas and facts for written reports	22
Form habits of not giving up, of repeated effort	21
Make brief outlines	21
Do not hesitate to memorize definitions, formulas, dates, provided you understand them	21
Prepare materials and place for work	20
Use habits of positive attack	20
Use dictionary often	20
Know parts of a book	20
Make use of aids to the thinking process as a whole—scientific attitude, etc.	20
Use all the varied library resources	20

IMPROVING STUDY

The Need for Improvement of Study Habits

Secondary school counselors are frequently asked to help pupils who are failing in their school work. In an analysis of the factors affecting success and failure, the ability to study effectively looms as a key point. Pupils often lack proper study habits and skills, and it becomes apparent that, in many cases, it is the lack of instruction more than the lack of ability that blocks effective learning.

One has but to observe many college students to realize that they, too, could profit from a more systematic program of methods of study. Furthermore, questionnaires to pupils themselves about their felt needs reveal that they wish that they had been given help on how to study.

In addition to individual techniques there is the factor of time-and-place. Study is customarily done—(a) at home, (b) in school study period, (c) in the classroom as supervised study. While each system has its advantages, the first two have serious limitations—the most serious of which is the fact that the teacher who is the guide for the lesson is not available to give direction or to clarify misunderstandings. With the change in home patterns and the encroachment upon "homework time" by such diversions as television and radio, schools are giving more attention to study under the direction of the school. Whatever the local policy about homework and study halls, three points are worth considering:

1. All pupils can profit by general suggestions on effective study.
2. The technique in studying one subject differs somewhat from that of studying another; therefore, special instruction needs to be given for each course.
3. Each teacher should employ supervised study procedures as a part of his classroom instruction.

Studying Mathematics

A pupil in a course in mathematics needs suggestions related to that particular subject. Some of the points of value to mathematics teachers are as follows:

1. *The work done should be meaningful.*
    a. The pupil should come to see the coherence between present and past assignments, between classroom work and life. (See section on "Articulation" in Chapter I.) Wherever applicable, bring out the social meanings.
    b. To gain understanding, the pupil should see mathematics as a form of communication; consequently, attention must be paid to the vocabulary peculiar to mathematicians. Familiarity with the conventional symbols is also necessary. It should be pointed out that many rules are just "rules of the game," matters of general agreement so that mathematicians everywhere will have a common language and a common mode of procedure. For example, in graphing, it is *common agreement* to say that the direction to the left on the X-axis is "negative."
    c. Develop the right mental attitude; try to see the significance of the work to be done in order to set a goal to be attained.
2. *Reading must be done slowly and with more care than narrative or descriptive reading which is common with some subjects.* Each word needs to be analyzed for its full meaning.
3. *Analyze the problem—find out what is wanted, what is given, what relationships exist.* Teachers can provide more realistic experiences by giving practice in problems that have wide applications.
    a. Make columnar arrangement of data, or
    b. Arrange data as related by formula, or

IMPROVING STUDY WITH GEOMETRY PROJECTS

   c. Make a sketch (especially in geometric problems) labeling the parts.
   d. Indicate what is known and what is yet unknown. Try to make drawings to represent abstractions.

4. *Form the habit of estimating the results.* Teachers need to give classroom practice on this habit and to help develop the attitude that manipulation is not the chief purpose of problems. Check the "common sense" of the answer later obtained. Outline the problem solution rather than complete all the computation involved.

5. *Form the habit of looking for suggestions and hints given by the author of the text.* Keep a notebook with a supplementary list of similar hints developed in class.

6. *Make notes when studying.* Especially list questions which you don't understand or about which you would like some more information.

7. *Write clearly.* Careless writing of digits and mathematical symbols will only confuse later on.

8. *Reread and rephrase the problem in your own words.*

9. *Develop the habit of looking for generalizations in the exercise,* for meanings beyond the mere words of the problem or the particular skill involved.

Specific problems are illustrations of general mathematical principles. List the principles in connection with each assignment. At first, this will need to be class-developed, but the pupil should be made increasingly self-reliant in this respect.

To avoid rote study, the pupil should have opportunity to work on mixed types of exercises rather than on one type only.

10. *Bibliography.* Teachers who want to work out more complete mathematics study plans for their own classes would do well to consider also the following sources of study discussion.

Almost any good text has some study hints—especially in connection with problem-solving. Some helpful sources are:

Butler, C. H. and Wren, F. L.,*Teaching of Secondary Mathematics,* Ch. VII. New York, McGraw-Hill Book Company, Inc., 1951

Brink, W. G., *Directing Study Activities in Secondary Schools,* Ch. XII. New York, Odyssey Press, Inc., 1937

Breslich, E. R., *The Technique of Teaching Secondary School Mathematics,* Ch. IV. Chicago, University of Chicago Press, 1937

Ligda, P., *The Teaching of Elementary Algebra,* New York, Houghton-Mifflin Company, 1925

Dadourian, D. A., *How to Study, How to Solve,* Cambridge 42, Mass., Addison-Wesley Press, Inc., 1949

Supervised Study

The recitation as commonly used is probably "the most fantastic of the vestigial remains of outmoded methods which still operate in the schools".[1] The previous discussion on how to study has indicated that learning can be improved by giving more class time to individual work under the direction of the teacher. Any good text on methodology will present some ideas on the technique of supervised study.

No one device should be used exclusively. Many teachers have found that improved learning comes from the following pattern. Spend only ten to fifteen minutes of the group's time in discussing the work to be done for the day. (This need not be daily when unit assignments are made.) The remainder of the period should be given over to individual work. The teacher moves freely about the room and plans to see each pupil at last once during the period. Whenever a common difficulty is noted, the teacher stops the class (if most are having difficulty) or takes the small group involved over to a side blackboard and goes over the explanation with them. When the nature of the work warrants, answer books may be available so that individual pupils may keep a check on their accuracy as they go along at their individual rates.

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[1] National Society for the Study of Education. *Forty-ninth Yearbook, Part I, Learning and Instruction.* Chicago, Ill., University of Chicago Press, 1950.

A LABORATORY APPROACH TO MATHEMATICS

Every good teacher of mathematics is concerned with the problem of making his course more meaningful. Yet it sometimes comes as a shock to a student in college mathematics to work with various instruments and to realize for the first time just what many of the abstracts really mean. The obvious thought occurs—Why have these experiences with objects instead of with abstractions been delayed so long? Why wasn't an opportunity to "learn by doing" presented with the introduction of various mathematical concepts in the secondary school?

A number of the more effective ways to eliminate the necessity for raising such questions are bound up in the laboratory concept—an approach different from that of the usual classroom routine.

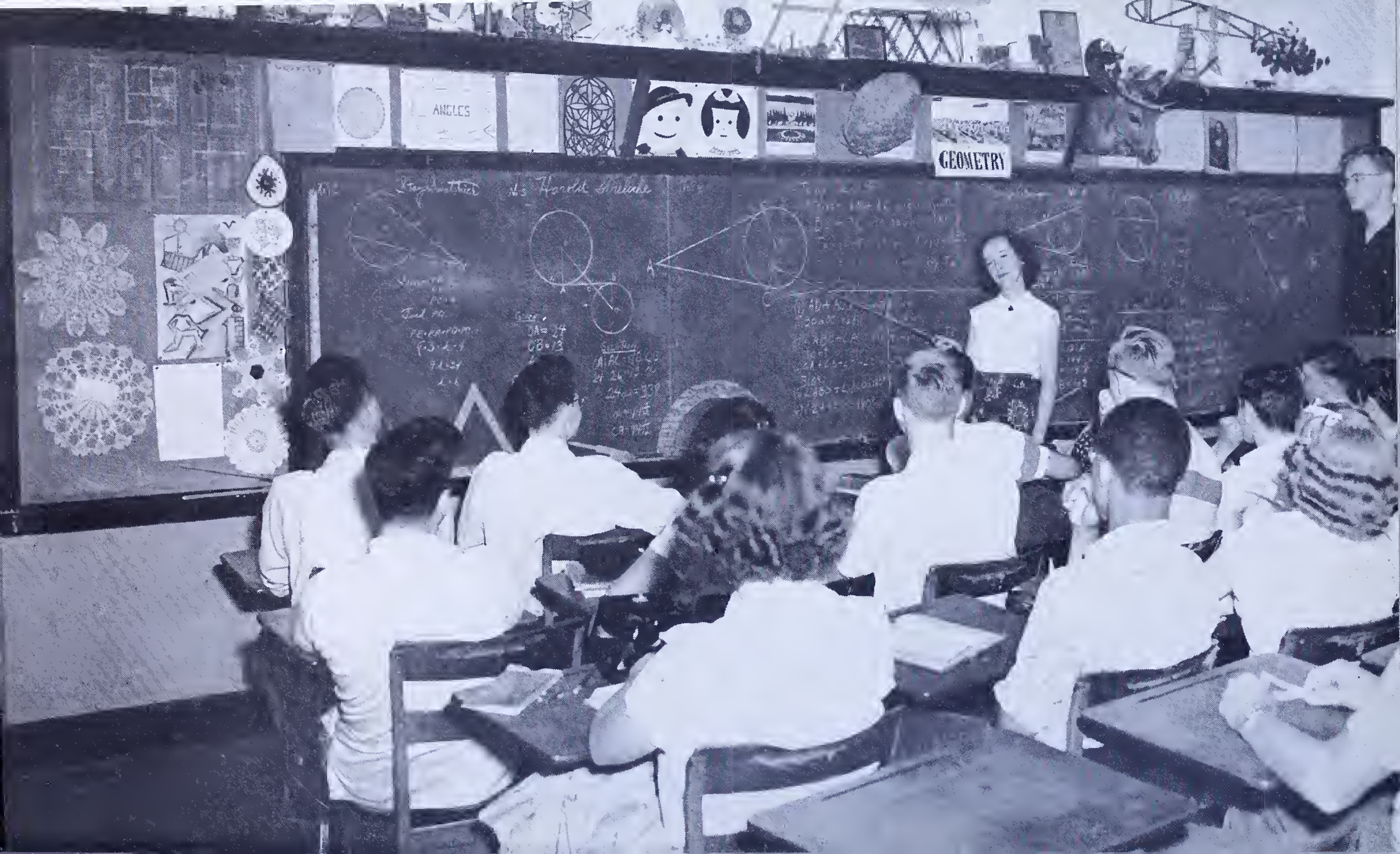
The Laboratory Concept

Most people are familiar with a laboratory as used in the natural sciences. The well-known Committee of Ten doubted the intrinsic value of the laboratory to the extent of declaring that time spent in it was equal to only one-half of that spent in the classroom. *However, it has been the experience of many pupils that learning is actually greater in the laboratory workroom.* Through objectifying ideas and principles, through demonstrations, through participation in various experiments, and through opportunities to gain skill in types of manipulation, the learners are able to give meaning to verbalisms. Why should these same learners be denied a similar opportunity to find life in mathematics? Just as the learner has found greater significance in seeing the amoeba as a living organism under the microscope, so also will he see "come to life" the vernier as an instrument for measuring when he has a chance to use a vernier in actual measurement.

One reason for so much difficulty in learning mathematics lies in the verbalistic quality of much teaching. Verbalism is the use of words without emphasis upon meaning and practice. It is little wonder that such teaching produces schoolboy boners such as "Parallelepipeds are animals with parallel feet" or "The Colosseum was epileptical in form." Imperfect and vague concepts are the usual outgrowth when teachers require pupils to learn various formulas and theorems without attempting to teach them with practice, meaning, understanding, and application to life situations.

Interest is also a factor in learning. The laboratory approach promotes more effective learning because it uses the following psychological principles:

1. **Pupils have more interest in immediate than in remote surroundings.**
2. **Pupils have more interest in the concrete than in the abstract.**
3. **Pupils want to learn how things work.**
4. **Pupils prefer their own activity in learning to that of being shown.**


ANGLES
GEOMETRY

In educational circles today there is a common tendency to talk about individual needs and abilities and to feel that little can be done about it in mathematics beyond some type of broad grouping. Certainly teachers can find little time for individual instruction (although they may be aware of the fact that it is the individual and not the group that learns) if they rely solely upon a question-recite technique. What is needed is a method that will allow teachers time to provide for individual differences within the class period.

Would a mathematics laboratory help hurdle these barriers to effective learning? An answer to this important question may be obtained after a thorough understanding of the meaning of "laboratory" is achieved in connection with the teaching of mathematics.

It must be realized that the laboratory concept as used here refers to more than a physical environment. It is an *atmosphere* in which problems may be worked out realistically through the medium of mathematics. It is an *approach* to the solution of mathematical problems. The same approach may serve for the solution of problems in other areas.

To most people the word "laboratory" conjures up a vision of a room with a demonstration table, numerous work benches, some wall charts, and an array of different types of apparatus. The physical limitations of a school building make the impracticability of setting up such special mathematics laboratories fairly obvious. There is already too much overcrowding, and several such special rooms would be required to serve the average-size mathematics department. Furthermore, one separate laboratory would not be feasible since it would not be available to more than one group at a time, thus restricting the basic requirement of flexibility for a true working atmosphere and ultimately resulting in as artificial a situation as that in the traditional classrooms.

Every mathematics classroom can and should be a type of mathematics laboratory. Wall charts, movable chairs which may be arranged in groups, mathematical instruments and models, and supplies such as rulers and compasses should be provided. The atmosphere established is the basically important thing, realistic working conditions must exist, and an interest in finding the solution of problems must be created. Fancy and expensive instruments are not needed in order to create the desire to find the solution. Satisfaction in finding it can be experienced through various "homemade" methods.

To reiterate, the mathematics laboratory is a concept and an approach; and as such the laboratory approach in every classroom offers many opportunities which will be of advantage to any teacher.

THE LABORATORY SPIRIT

The Equipment

On the assumption that a regular classroom as well as a special room can present a laboratory-for-learning environment, what aids can be mentioned to help develop that atmosphere? Lists of materials have been compiled by writers on mathematical aids. Articles in *The Mathematics Teacher* and in *Mathematics at Work,* the report of the Eighth Annual Mathematics Institute at Duke University, enumerate various instruments and articles. It would remove some of the thrill and hamper the basic experimental attitude if this bulletin should prescribe lists of items as "musts" for the laboratory environment. However, most teachers would probably consider the following categories, select from them, and then adapt aids in terms of the purposes of the particular course, the nature of the pupils in each class, and the projects under consideration:

1. *Models*—Some may be purchased, but the most effective will be those developed by the pupils themselves. Various solids such as a spherical blackboard, conic sections, etc., may be constructed.
2. *Visual Aids*—Filmstrips, slides (class-made), projector for films to be borrowed, rented, or purchased. Consider three-dimensional drawings to be used with special glasses worn by the observers. (See *18th Yearbook of the National Council of Teachers of Mathematics* for other suggestions concerning multi-sensory aids.)
3. *Library Materials*—Supplementary texts in mathematics, astronomy, physics, navigation, and other arts and sciences making use of mathematics. Current magazines of wide variety. Books revealing the role of mathematics in the growth of civilization (including biographies). Books of mathematical games and recreation. Files, with pupil contributions, of clippings, jokes, pictures of mathematical import.
4. *Wallboards*—The usual blackboard with some sections ruled into graphs; sections of wallboard suitable for using thumbtacks for posters, clippings, and pupil projects.
5. *Blackboard Tools*—Meter sticks and yard sticks, colored chalk (for special emphasis), blackboard compasses and protractors, 30 x 60 degree and 45 x 45 degree triangles, T-squares.
6. *Instruments of Applied Mathematics*—Slide rule (large, for demonstration purposes), balances, pendulum, surveying equipment, the plane table, pulley-lever-gear machine, and numerous other objects culled from the learners' environment. Precision instruments may be used where groups are interested in industrial mathematics.
7. *Construction Materials*—An endless field for a "new world" of mathematics can be created from bits of string, wood, cardboard, plastic, and metal to challenge the ingenuity of the participants

in model building and in making "machines" to demonstrate mathematical laws. Eyelets, punches, glue, scissors, razor, etc., should be on hand.

8. *Teacher Aids*—Workbooks and worksheets. These have certain limitations but serve practical purposes for some learners—especially for the maintenance of skills and for attention to remedial work. Resource units: See Jones, Grizzell, Grinstead, *Principles of Unit Construction,* McGraw-Hill Book Company, for help on the unit approach; Thut and Gerberich, *Foundations of Method for Secondary Schools,* McGraw-Hill Book Company. Innumerable devices such as a homemade, demountable parallelogram.
9. *Community Resources*—Advertisements with mathematical forms. concepts, notations, income tax blanks; insurance applications; sales slips; budget plans; time-payment forms.

Organizing Learning Experiences

The laboratory concept is more than a classroom environment. Indeed the community is regarded as a part of the learning laboratory. A teacher with the laboratory concept is greatly concerned with the discovery and utilization of many community resources. Community-relatedness is not restricted to field trips.

Fundamental also is the organization of subject matter which provides for as many types of functional activities as possible. While many are aware of the psychological advantages of presenting material through significant units, it is important to note here that the day-by-day, page-by-page assignment should give way to long-range assignments which incorporate a wide variety of suggested functional activities. At present, general mathematics offers great opportunity along this line, but if the approach is helpful to the learner, the more traditional offerings should be reorganized to make the best use of the psychological advantages of the unit. Especially valuable for a working environment are resource units from which material may be drawn as occasion demands.

DEVELOPMENT OF UNITS

Many problems of education will be resolved when we can give a conclusive answer to the question "How do we learn?" Of the several prominent theories relative to learning, there is considerable evidence to support the conviction that we learn by wholes. The child is a unit biologically; he is a unit psychologically; and as a social being his real experiences are made up of many interrelated parts. The only break in the picture seems to be the child's school experiences, where some things tend to be fragmented.

The section of Chapter I on "Articulation" is an attempt to give greater unity to the school environment. Of as great value is the growing educational practice of organizing these learning experiences into related wholes, known as units. It should also be remembered that the unit approach enables the teacher to provide better for the condition that differences in individual behavior are due to differences in ability to learn.

Units range in types from the purely subject-centered to the life-problems-centered. Some texts have tried to "jump on the bandwagon" by calling their chapters units. To be sure there is unity within subject fields, but this is often the logical unity of organized race experience. However, the logical unity is not necessarily psychological unity to the beginner. The adolescent learner is one who learns by *his* experience, who sees the concrete before the general, who moves from the near to the remote. There is the danger that teachers, as mature persons who have an overview of the entire field, may fail to realize that, while the logical is psychological to them, the youth may learn things in a different order.

Units may also be classified as to purpose and as to development. Thus we hear of learning units, teaching units, and resource units. This chapter includes examples of resource units, the use of which is described under the heading of "Characteristics of a Resource Unit." However, any educational outline, for a unit can be but an outline and not the actual learning, consists of three essential parts: (1) purposes, (2) educational experiences to carry out these purposes, and (3) evaluation—plans to determine how well the purposes are being realized. The individual teacher of mathematics should consider these phases whether he builds the unit himself, helps his pupils develop the unit, or works with other teachers.

Purposes

These are the goals that are the reason for the educational effort. A well-organized unit has one major purpose often spoken of as the Central Objective. Usually it is stated in terms of an *understanding* or an *ability* of the learner to do something. To work toward this central objective the learner has to practice certain behaviors, develop certain skills, and build up certain understandings. These are but helping outcomes and are called Contributory Objectives. There are also some expected by-products of the learning efforts, and these may be listed as Concomitant Outcomes.

For example, a subject-centered unit might be entitled "The Ability to Factor." Although this is a goal, it is hardly a purposeful one in itself; it is rather a tool to enable the pupil to handle certain problem situations. As such it is a contributing objective. When it is made the main objective, it tends to become little more than an exercise in manipulation.

Activities

These are the suggested learning experiences which will help the individual move toward the stated purposes. Even though it may be expedient to set up and outline units ahead of time, pupils cooperatively may work out many approaches under the guidance of the teacher. Such planning develops pupil purposes beyond getting a mark and pleasing the teacher.

Since learning is individual, it is important to suggest a wide variety of activities. They should be suitable for both group and individual work; they should appeal to all types of sensory approaches. The shortcoming of most units is that the activities or learning experiences are too limited. It should be emphasized that the list of activities be suggestive since not all may be appropriate for the immediate class or for the unit emphasis which develops as the class works.

a. Preparatory or Initiatory Activities

Here the purpose is to orient the group to the unit, to motivate for purposeful activity. This may come through such devices as pretest (discussed later), a teacher presentation, a group discussion, an outside speaker.

b. The Developmental Activities

This large group of pupil experiences will enable them to gain the desired skills, habits, attitudes, understandings and behaviors. Since the ultimate goal of the school program is to produce individuals who will live with satisfaction to themselves and with profit to society, it is important to provide for practice in group processes as well as in individual expression and investigation. However, the activities should not be busy work but should be aids in the attainment of at least one of the objectives.

c. Culminating Activities

Since, unlike the conventional assignment, all will not be working on the same thing at the same time, there is some danger that the learning will be unorganized and lacking in the necessary unity. Consequently, Culminating Activities need to be included in the planning. Exhibits, reports, panel discussions, individual notebooks, debates, playlets are among the common devices in this group of activities. A review followed by a test is suitable in many situations.

Evaluation

Plans must be made to determine what growth has taken place, how far it has gone, and in what direction. There should be a definite attempt to check on every one of the objectives. It is often helpful to start the unit with a pretest: (1) to discover what the class already knows so that the teacher may build upon that, (2) to find what the class either does not know or misunderstands so that emphasis may be placed on these areas, and (3) at the end of the unit to estimate the improvements that have taken place. Paper-and-pencil tests are but one method of evaluation. Observation, projects and reports, and self-evaluation are among other methods. (See Chapter IV on "Evaluation" for details.)

CHARACTERISTICS OF A RESOURCE UNIT[1]

I. *Why is a resource unit prepared?*

1. A problem which has a useful purpose in the life of the pupils is suggested.
2. The teaching-learning unit which is suggested is related to the stated objectives of the school.
3. Means of meeting the pupils' needs are suggested.
4. Learning is guided away from textbook domination.
5. The democratic method by means of which the teacher and pupils work together, as in planning learning activities, is facilitated.
6. Provisions are suggested by which the pupils in studying the problem utilize the skills and abilities in pertinent fields, such as English, social studies, and the fine and practical arts.
7. Activities and materials are not prescribed but are suggested, and aid, therefore, in stimulating the teacher's initiative in planning a teaching-learning unit.
8. Guide lines as means of introducing a teaching-learning unit and adapting the resource unit to a teaching-learning unit are provided for the work of the teacher in planning such a unit.
9. Recent research materials in fields such as science which are pertinent to the study of the problem are synthesized.
10. Information, such as reading material, which will enlarge the teacher's background about the problem is given.
11. The time of the teacher in planning a teaching-learning unit is conserved.

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[1] From a doctoral study by M. Elaine Duffy, University of Pennsylvania, 1951.

II. *Who plans and prepares a resource unit?*
  1. It is planned by a teacher.
  2. It is planned by a group of teachers.
  3. It is prepared by a group of teachers with a consultant.
  4. It is prepared by an expert.

III. *What does a resource unit contain?*
  A. Objectives for a teaching-learning unit are:
    1. Based on the interests of the pupils
    2. Socially desirable
    3. Developmental
    4. Spread over a wide range of abilities
  B. Subject matter for a teaching-learning unit includes:
    1. The scope of the problem used in a teaching-learning unit
    2. One or more teaching-learning units
    3. Material from fields pertinent to the study of the problem
    4. Emphasis centered in one field, for example, social studies
  C. Activities for a teaching-learning unit are:
    1. Selected so as to attain the broad, general objectives of the teaching-learning unit
    2. Experiences which contribute to the all-round development of the pupils
    3. Without attention to conventional subject matter lines
    4. Inclusive of varied activities such as listening, observing, and experimenting
    5. Activities which provide for a range of achievements and abilities
    6. Activities which provide for a range of interests
  D. Means of evaluation for a teaching-learning unit:
    1. Means of determining the needs of pupils
    2. Means of determining the interests of pupils
    3. Means of evaluating the attainment of the broad general objectives
    4. Means of evaluating the attainment of specific objectives
    5. Opportunities for pupil participation in evaluation
  E. Reference lists pertinent to a teaching-learning unit:
    1. Sources of inexpensive and free materials
    2. Annotated lists of reading materials for the pupils
    3. An annotated list of reading materials for the teacher
    4. An annotated list of audio-visual aids
    5. An annotated list of community resources

IV. *What factors determine a resource unit?*

1. Preparation in a school in which there is a need for curriculum development
2. Preparation in a city in which there is a need for curriculum development
3. Preparation in a state in which there is a need for curriculum development
4. Preparation on a national basis in a field in which there is need for reorganization
5. Recognition of an appropriate philosophy of education for existing conditions
6. Development of the community for problems for a teaching-learning unit
7. Analysis of the needs of the pupils for problems for a teaching-learning unit
8. Analysis of a master plan for the study of the problem for a teaching-learning unit
9. Revision of the plan upon the basis of utilization of the resource unit in the development of a teaching-learning unit

V. *How is a resource unit utilized?*

1. Suggestions of activities, means of evaluation, and references are used as a reservoir by the teacher in developing a teaching-learning unit.
2. Guide lines for the work of the teacher in planning and evaluating the teaching-learning unit are provided.
3. A basis for cooperative teacher-pupil planning is given.
4. Provision for day-to-day teaching procedure is not given.

VI. *What are the contributions of a resource unit to the improvement of the teaching-learning activity?*

1. A problem involving a significant area in living in society
2. A problem involving a significant area of living in the work of the school
3. The needs of the children in planning a teaching-learning unit
4. The interests of the children in planning a teaching-learning unit
5. The study of a problem which is practical under school conditions
6. Aid in the unification of learning experiences of the pupils
7. Aid in the provision for the continuous growth of the pupils because one center of interest is not assigned to one grade level

8. Prepared for more than one particular class
9. Prepared for more than one specific grade level
10. May be used in more than one teaching-learning unit
11. More materials than one teacher can use with one class
12. Leads to other units of learning are suggested

ILLUSTRATIVE RESOURCE UNIT

HOW CAN MATHEMATICS HELP BOYS AND GIRLS TO USE THEIR LEISURE TIME PROFITABLY?

I. Introduction

A. What Is a Resource Unit?

A resource unit is a source of suggestions for objectives, problem analyses, questions, activities, teaching aids, and means of evaluation developed by a teacher or cooperatively by a group of teachers. It is concerned with a problem of life which provides for a range of children's interests and abilities. A resource unit is drawn upon selectively by the teacher in planning learning experiences for the children. Nothing in the resource unit is prescribed; everything is suggested.

B. Selection of the Problem

The needs of the pupils which are to be satisfied are listed. Examples:

1. *Need to develop salable skills*
    a. Learning to work effectively with others
    b. Becoming familiar with the vocational opportunities in the community surrounding the home and school
2. *Need to maintain health and physical fitness*
    a. Participating in some form of light labor, such as landscape work, lawn mowing
    b. Enjoying exercise such as team games—baseball, football, basketball
3. *Need to understand rights and duties of citizenship in a democratic society*
    a. Achieving status with fellow students through increased competence in profitable use of leisure time
    b. Increasing understanding of consumer relationships
4. *Need to budget time by choosing a balanced variety of activities*
    a. Choosing activities to develop personally satisfying and socially valuable skills
    b. Choosing activities which serve to promote enjoyment and profit
5. *Need to develop respect for other persons*
    Planning, sharing, and working together to achieve group progress

C. Objectives of This Resource Unit

1. Ability to select a profitable leisure time activity
    Understanding the factors considered in its selection
2. Ability to participate in a well-selected leisure time activity
3. Ability to apply mathematical skills to situations involving profitable use of leisure time
4. Appreciation of the value of leisure time activity
5. Attitude of personal responsibility

II. Analysis of the Problem

A. STATEMENT OF THE PROBLEM

How can mathematics help high school boys and girls use their leisure time profitably?[1]

B. PROBLEM DEFINED

This problem includes the monetary and nonmonetary gains which boys and girls can receive in their leisure time as a result of mathematics learned in school. Realizing that pupils have time after school, over week ends, and during vacation, in which to choose different kinds of activities to occupy their free time, the committee selected this unit to give suggestions for the wise use of that leisure time. Activities carried on in leisure time may provide monetary or nonmonetary gains or both. Those activities from which the pupil receives no financial compensation but a feeling of satisfaction are listed under *nonmonetary gains*. These gains result from the solution of problems relating to interests and needs and occur through recreational activities. Those gains which are in the form of financial compensation for work done or service performed are called monetary gains, and the activities engaged in are listed under *monetary gains*.

C. LEISURE TIME HAS BEEN DIVIDED INTO THREE PARTS: AFTER SCHOOL, WEEK ENDS, AND VACATIONS

Suggested activities are listed under each part according to nonmonetary gains and monetary gains. There is a separate list of suggested activities under Vacations, as shown below, since there are various extra jobs available to pupils during vacation time which are not available over week ends or after school.

The suggested approaches are divided into three parts: (1) Orientation, (2) Carrying Through, and (3) Evaluation. The orientation describes how the teacher and pupils can arrive at the choice of an activity, their reason for selection, the limitations involved, and the procedure to be used. Carrying Through gives a general picture of how any of the activities listed are to be developed. Evaluation is necessary to determine the direction and the extent of the process.

The following subdivisions under Vacations apply as well to the other main divisions, After School and Week Ends.

VACATIONS

Introduction

1. Nonmonetary Gains

   Recreational activities (laying out a tennis court, baseball diamond, football field, etc., choosing playground equipment, considering the placing, time, and money involved in laying out various recreational fields.)

2. Monetary Gains
   a. Clean up jobs (windows, lawns, sidewalks, etc.)
   b. Lawns (mowing, trimming, etc.)
   c. Farm and orchard work (picking fruit, vegetables, construction work, etc.)
   d. Retail selling (magazines, newspapers, etc.)
   e. Odd jobs (baby sitting, repair work, etc.)
   f. Additional suggested activities for vacation time
      Post-office work
      Hotel (waiter, bellhop, elevator boy, assistant cook, etc.)
      Swimming pool (attendant, refreshments, clean up, etc.)
      Christmas or Easter jobs (post office, clerk in stores, etc.)

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[1] The importance of this theme is underscored by the appearance in 1950 of the Life Adjustment Booklet, *Enjoying Leisure Time*. This booklet with an accompanying teacher's manual is published by Science Research Associates, Inc., Chicago, Ill.

3. Monetary or Nonmonetary Gains
    a. Landscape gardening (laying out and developing a garden and grounds)
    b. Repair work (radio, gadgets, playthings, etc.)
    c. Hobbies (repairing radios and other things, making small articles, building things)

III. Questions and Activities

A. SUGGESTED QUESTIONS FOR DEVELOPING INTEREST AND FOR GIVING DIRECTION TOWARD AN ACCEPTABLE SOLUTION OF THE PROBLEM

1. How can I earn more money?
2. At how much have I valued (or would I value) my time and labor?
3. What part-time jobs are available in my community?
4. For which of these jobs would I be eligible?
5. What would be the most profitable disposition of my earnings, i.e., should I spend my earnings for goods as soon as earned, should I deposit the earnings in a bank, should I purchase insurance, should I invest in Government Savings Bonds, should I invest in commercial securities, etc.?
6. Should my present allowance (if any) be reduced if I begin to earn money of my own? If so, will the difference between my present allowance and the sum of my prospective earnings and reduced allowance be sufficient to justify the expenditure of my time and energies?

B. SUGGESTED ACTIVITIES (The ones listed are but a few examples of many other possibilities.)

1. Out of School
    a. Clean up jobs (windows, lawns, sidewalks, house, store, office, etc.)
    b. Plan and construct a baseball field. Lay out the diamond and construct the grandstands. (Group activity)
2. In School
    a. Handle the finances for school dances, plays, and other functions.
    b. Study the finances involved in the school's athletic program. Do some sports cost too much?

C. READING ACTIVITIES

1. Select from the school library or from the community library, a book which will improve proficiency in the field in which the pupil is now working (or in which he has already attained some proficiency).
2. If there are periodicals concerning his field of interest, the pupil should examine issues regularly for articles which might improve his proficiency. Each pupil should subscribe to at least one such periodical if possible, or purchase it regularly at a newsstand.

D. EXHIBITS, PICTURES, CHARTS, MAPS, DIAGRAMS, DRAWINGS, ETC.

Preparation of graph to show annual interest earned by each dollar—(1) deposited in the bank, (2) invested in life insurance, (3) invested in Government Savings Bonds, (4) invested in a profitable commercial security, (5) invested in average security recognized by New York Stock Exchange (teacher must secure figures for this from some reputable index) for each year over the last 20 years. Preparation of graph to include the above information adjusted to the purchasing value of the dollar for each year.

E. Lectures and Interviews

1. Student prepares a short talk for the class about the field in which the boy or girl is interested or engaged.
2. Lecture by a representative of community business, such as the local bank official, on benefits of and procedures in depositing earnings in banks and purpose of banks in general (including the making of loans to good risks).

F. Observation Trips

1. Visit a factory in the community. Note how important a knowledge of mathematics is to various employes of the factory.
2. Visit a scientific museum. Point out how many of the improvements and inventions shown could not have been accomplished without the aid of mathematics.

G. Research

1. Examine the classified advertisements in one newspaper every day for a given period of time to record all jobs available for high school pupils and to prepare a summary of the fluctuations in demand for each type of position from month to month.
2. Study of hall and stair traffic in the school building by a committee of pupils. Could the movement of students from class to class be accelerated by better planning?
3. Organize a group to study the sale of newspapers, refreshments, etc., which usually accompanies an athletic event. Could students form an organization which could serve such events more profitably than is now the case under individual competition?

IV. Evaluation

Sample Questions on Areas Under Consideration

1. Is the student engaged in a profitable leisure-time activity? Has his status in this regard changed since the beginning of this course?
2. Is the student's selection of the activity commensurate with his aptitudes as indicated in school records of his achievement in aptitude tests?
3. Does the student keep a neat and accurate record of his earnings, disbursements, interest, investments, etc.?
4. What is the student's average rating on responsibility as estimated by all his teachers? Has this rating increased in the past year?
5. Is the student enthusiastic about using his leisure time profitably? What do his parents think of this? His employers?
6. What does the student's library card reveal as to his profitable use of leisure time? What books does he read? With what frequency? Has he undergone any change in this regard?
7. Observe and note the questions posed by the pupils throughout the course and particularly at the time when speakers are brought to the class.

INDIVIDUALIZING INSTRUCTION

One of the assumptions behind the "Master Plan," which follows in Chapter III, page 68, is that learning is an individual process; consequently, every effort should be made to discover the individual needs of each pupil and then utilize all techniques possible to adapt the instruction to the individual. This process is easier to discuss than to put into action since, carried to its logical conclusion, it would mean thirty different lessons if a teacher has thirty pupils in a class.

When the class is conducted on the basis of a daily recitation and assignment for all, individualization is difficult. The development of long-range assignments and the use of supervised study (discussed on page 44) allow for much more flexible classroom procedures.

Some schools may reduce the range of ability within any one class by employing some form of homogeneous grouping. But even here the pupils will not be homogeneous in all respects; so the need to individualize instruction is still present. Whatever approach is used, it is necessary for the teacher to know his pupils—to be familiar with the background, the test indications of ability, the current interests, the future plans of each one. When the classroom is run as a learning laboratory, this person-to-person basis is easier to establish. But there still remains the challenge for administrators to work out plans so that class size and teaching loads will be reduced.

Differentiated Tests

Although there is a content basic for all pupils. there is also considerable material in each course which cannot be effectively learned by all because of wide variations in individual abilities. To overcome this, it is suggested that all pupils should not be held responsible for the same work.

It is advisable to divide the class into two, three, or four groups. The 3-group plan has divisions adapted to the slow, average, and fast learner. There will be some at the lowest end of the slow group who will still require separate instruction from the rest of the group, but this comes under the heading of individual remedial work.

It must be remembered that the class should be prepared for this division. It is suggested that it be explained to the class that

1. Passage between groups is possible, especially from a lower to a higher group.
2. The emphasis is on the pupil's achievement in relation to his own ability and not other pupils' achievement.
3. Mistakes in grouping are possible and will be corrected when found.

A few advantages of this type of course and testing are listed here:

1. Each pupil works at his own speed; none is rushed.
2. Each pupil works to capacity. It is always possible to work problems of another group for extra credit and for progress.
3. The whole class will have opportunities to master the simple basic content.
4. Since each pupil is kept busy, discipline problems will be minimized.

In appraising pupil progress under such a setup, the teacher may use three separate tests or a test for the basic part, plus work for the average pupil, plus work for the fast student. Or the teacher may use the contract method, in which the course or test is in terms of minimum, average, and maximum work. An example of this type of test in the field of stocks and bonds follows. The pupil takes the test in the group of which he is a member.

Contract Method Test

Group I. (*Slow*)
  a. How does the bond differ from the stock?
  b. What is the purpose of stocks and bonds?
  c. What is the current price on these stocks?
  d. What is the current price on these bonds?
  { Stocks and bonds are available for students' use.

Group II. (*Average*) Above work plus
  a. What is the current yield on these bonds?
  b. If you held the bonds for two years, how much interest would you receive?

Group III. (*Fast*) c and d of Group I, plus a of Group II, plus
  a. Which is a safer investment and why—stocks or bonds?
  b. Which stock is the best investment in your opinion, and why?

Characteristics of Individualized Work

1. Seats are arranged to form groups when problem work is assigned, and assistance is given to slower pupils not only by the teacher but by other pupils as well.

2. When basic principles are taught to the group as a whole, the presentation is such that it will reach most of the slower pupils, but further explanation (to small groups) will be necessary at various times in the course of the work.

3. Differentiation of assignments in problem work is the next step. Such differentiation may take place in the form of a *minimum* assignment for all pupils, with a *maximum* assignment included. In a real sense there is no *maximum* since a teacher should not limit a pupil's endeavor but should encourage pupil initiative to find enrichment activities. The term is used here to designate three levels of achievement—minimum, average, maximum—with

the understanding that the satisfactory completion of any is a necessary requirement for attainment.

4. Quality not quantity should be the key in individualizing assignments. In the minimum assignment work the more capable pupils should not be expected to complete all the work before going on to the more challenging exercises. For example, if there are twelve problems in the minimum assignment the teacher should "spot" the basic types and have the better pupils do only those which will assure both the pupil and the teacher that the fundamental concepts are understood. Then the more capable pupils should proceed with appropriate work from the top-level assignment.
5. Pupils interested in some particular outside or shop task may bring in related problems to be solved by algebraic means, and a group may attempt to find the solution of such problems in place of the more stereotyped ones usually found in textbooks. This search for practical problems is encouraged even in beginning work with simple equations and formulas.
6. Each pupil proceeds to work individually or with a group on the problems which will develop the desired mastery of fundamental principles or skills, an assignment which will cover a period of time extending from several days to several weeks, depending on the topic. Cooperative assistance among pupils is encouraged, and the teacher is in the position of a resource person or consultant to the pupils.
7. Homework, eventually, will be practically self-assigned, with the completion of a sufficient number of tasks to produce a feeling of mastery of the topic on the part of the pupil and satisfaction in attaining the goal. At first, however, homework will be intelligently planned by the teacher with the cooperation of the pupils themselves. This is a step toward self-reliance. Tests which have been duplicated and are available for particular units within a given topic are taken by each pupil as he completes work in these units, and a simple graph of his progress may be kept by the individual. This is commendable since the graph measures personal achievement.
8. Within the allotted time for completing the assigned topic the more capable pupils will generally be found to have completed satisfactorily the assignments for their group, with the slow pupils generally finishing assignments adapted to their rate of progress. There may be a few who remain consistently behind even in the minimum work. These pupils may be found to require guidance

into another phase of the mathematics offerings of the school, or at least they will need individual help on how to make more effective use of their efforts.

9. From among the best pupils in the group certain ones may be given the opportunity to teach a lesson during the course of the semester. Plans for this lesson are developed by the teacher and pupil cooperatively, and the results will generally be found to justify the time, work, and ingenuity required in its preparation.

10. A class such as the one described will not always be a "quiet" class, nor will the chairs always be found arranged in the conventional fashion. They will be grouped in various ways at various times, and the visitor entering the classroom may have difficulty identifying the teacher immediately. The latter may be a member of one of the groups at that moment, filling the role of leader, acting as a guide in attempting to help each pupil develop his own particular abilities in the mathematics work. Such attention to individualizing instruction will go far to promote a sense of achievement in even the slowest member.

AN ILLUSTRATION OF INDIVIDUALIZED INSTRUCTION

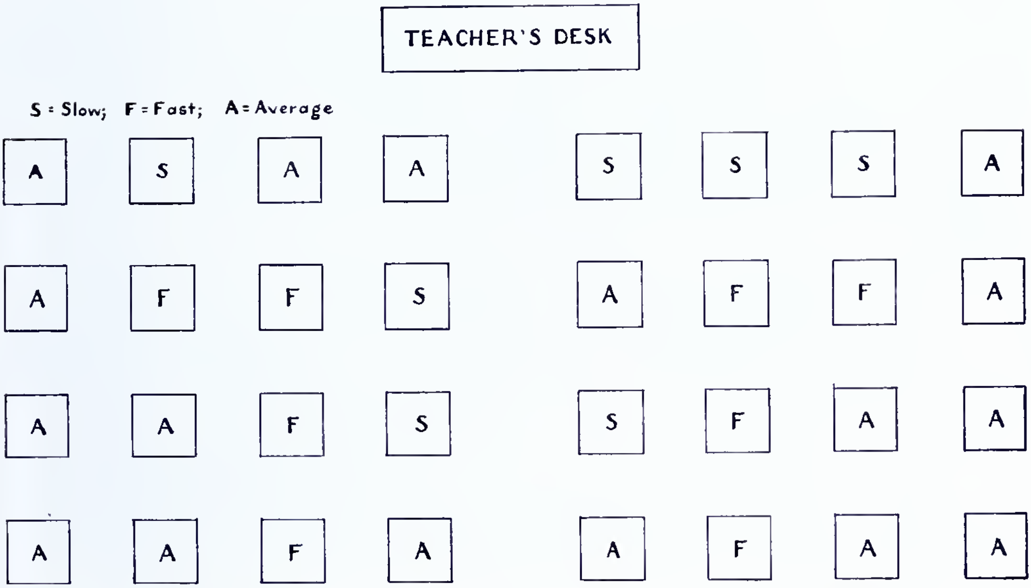
Individualized Instruction

The following description of Individualized Instruction with a Beginning Algebra Class is an illustration of what can be done to individualize instruction in the average classroom in a secondary school. It describes a pattern followed with success in an actual situation.

The teacher is scheduled for a class of thirty-two pupils in beginning algebra (Algebra I). He has a prescribed course of study outline but recognizes its flexibility. In consideration of the minimum requirements of the outline for the term, the teacher then plans with the pupils the specific learning activities.

The first responsibility of the teacher is to know each pupil as thoroughly as possible, and detect as early as possible physical disabilities, such as poor vision or hearing, which may necessitate seating a pupil in a particular position.

The approximate potential ability for algebra is determined for each pupil through study of his record of achievement in arithmetic, through some form of prognostic test, and through direct observation of classroom work during the first few weeks of the term. (These steps are preliminaries but may take one or two weeks to cover.)



TEACHER'S DESK

S = Slow; F = Fast; A = Average

A	S	A	A	S	S	S	A
A	F	F	S	A	F	F	A
A	A	F	S	S	F	A	A
A	A	F	A	A	F	A	A

A SEATING CHART TO FACILITATE INDIVIDUALIZED INSTRUCTION

A seating chart is then made up and the pupils who are apparently slower are arranged in positions of accessibility to the teacher and to more apt pupils. The seating chart is flexible, with changes made whenever the individual's progress or the work at hand warrants this. By the end of the semester there will be no further use for the chart at all.

NOTES

USING COMMUNITY RESOURCES

Another important phase of horizontal articulation is the relating of classwork to the local environment. There are many obvious ways to utilize community resources in teaching mathematics, such as having the class or a committee visit the town hall, special industries in the community, and the town library. A listing of possibilities for vitalizing mathematics instruction follows:

1. Select films as a medium of communication to vitalize the role of mathematics at work. These may be obtained at college or university film libraries in some areas. Seek cooperation of local theater managers in selecting "shorts" to be incorporated with programs especially adapted to teen-age audiences.

2. Make mathematics meaningful by showing how it can be fundamental to the beautification of school and home grounds. Measurement of walks, hedges, fences, buildings, and transformation of this material to scale drawings will lend realism to the world of numbers.[1]

3. In some rural areas the interest of some boys in solid geometry may be aroused by the study of the honey comb—volume, dihedral angles, etc.

4. A trip to the local airport where a competent instructor will show the various instruments and outline their principles. This will bring to the fore the realization of different types of measurement as well as the need for accuracy in computation. This is especially true in a discussion of navigation problems.

5. A twofold objective may be achieved by studying gambling machines. Here is a chance to make concrete the laws of probability. It will also tend to discourage young people from tossing away money when they prove that "you can't win."[2]

6. Go to an inspection garage to learn how speed affects distance in stopping, how the elevation of headlights affects the area of the beam. These situations in themselves may be physics problems, but they utilize the study of the formula and refer to the social implications of driving.

7. Talks by professional men such as engineers, insurance men, and bankers on the part mathematics plays in their particular field, will show the everyday application of mathematics in various phases of community life.

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[1] For a report on teaching procedures of this type, see the Keene Teachers College (N. H.) experiment as outlined in *School Science and Mathematics*, Vol. XLVI (April, 1946), pp. 313-317.

[2] See "The Mathematics of Gambling," by Ernest Blanche, in *School Science and Mathematics*, Vol. XLVI (March, 1946), pp. 217-227.

8. Conservation, whether it be a study of the effects of erosion and floods or the value of garbage, offers limitless areas for relating the problems of collecting and evaluating statistical data to community service.[1]
9. Get mail order catalogs from Sears, Roebuck and Company or similar organizations and make out orders to show the value of quantity, quality, size, mailing charges, money orders, personal checks—all mathematical usages which are common in adult life.
10. A visit to a lumber yard will afford an opportunity to learn what computations are made in selling building materials.
11. Most homes have a gas or an electric meter or both. Visiting an electric plant and keeping a record of meters at home will be valuable experiences.
12. In most communities there is a bureau of weights and measures. A visit to the bureau will be an excellent field trip for a mathematics class studying the metric system.
13. Some pupils may be planning to become teachers. Valuable experience can be given them by having them work with the teacher on school registers. This will not only give them practice in functional mathematics but will also help develop a better understanding of an important responsibility of teaching.
14. In trigonometry classes an understanding of the meaning and uses of trigonometric principles will be vitalized by bringing in someone familiar with aviation. The use of such devices as the sextant and the plotting board can be demonstrated and explained.
15. Construction of Christmas designs affords experimentation with solids. Some communities offer prizes for the most original and most effective Christmas scenes.

The use of units, especially of the resource unit type, affords an excellent opportunity to increase articulation. The unit is discussed in detail in Chapter II, page 49.

"What is really important is that each thing we teach the child be integrated with his own life and his experience. It must fulfill some need; it must give expression to some impulse; it must contribute to some thinking. It must, in short, have significance."[2]

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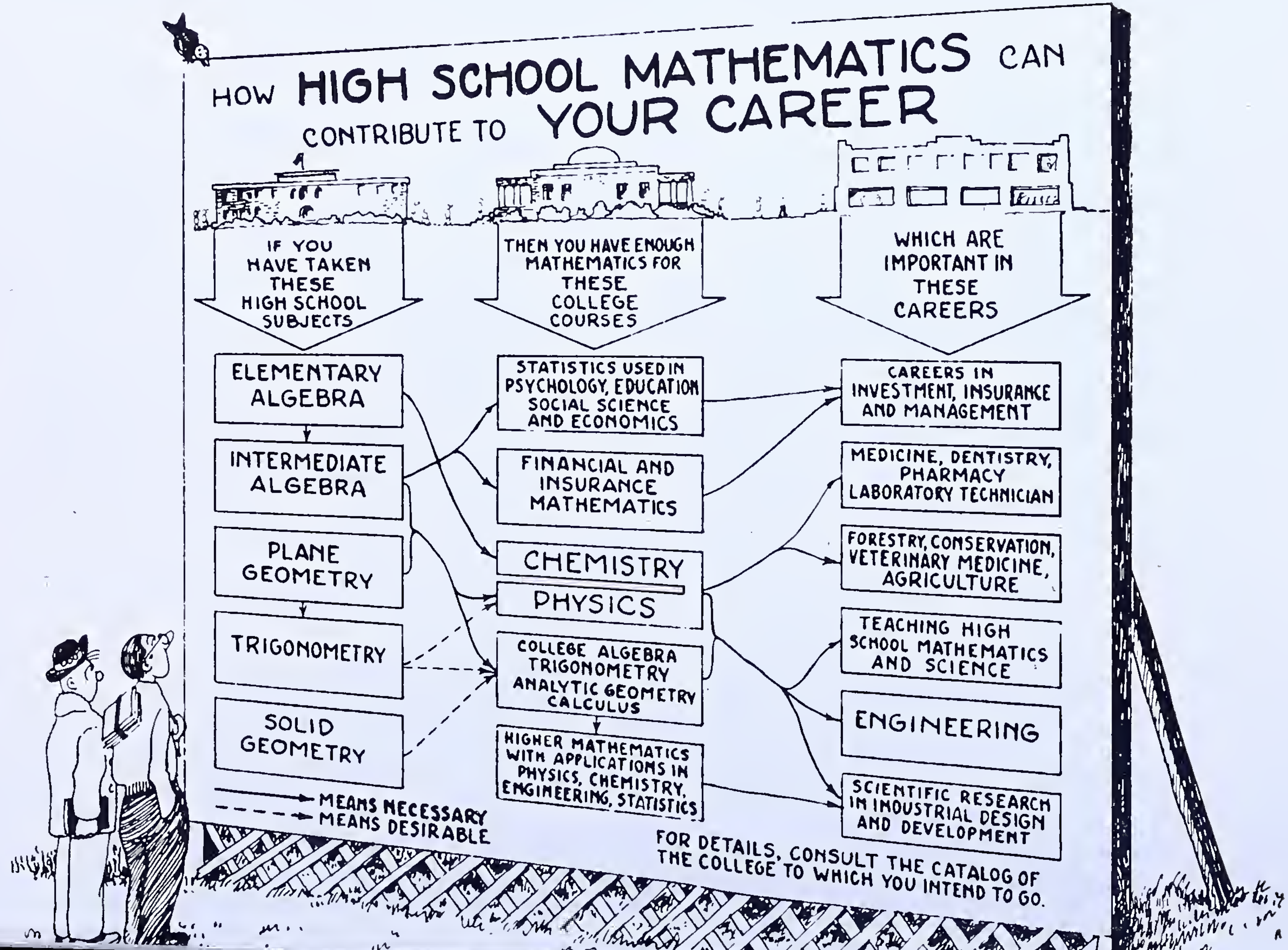
[1] See "Conservation and Mathematics," by H. P. Fawcett and H. J. Barcus, in *School Science and Mathematics*, Vol. XLVI (June, 1946), pp. 505-516.

[2] W. D. Reeve, "Mathematics and the Integrated Program in Secondary Schools," *Mathematics Teacher*, Vol. 30 (April, 1937), p. 165.


HOW HIGH SCHOOL MATHEMATICS CAN CONTRIBUTE TO YOUR CAREER
IF YOU HAVE TAKEN THESE HIGH SCHOOL SUBJECTS
THEN YOU HAVE ENOUGH MATHEMATICS FOR THESE COLLEGE COURSES
WHICH ARE IMPORTANT IN THESE CAREERS
ELEMENTARY ALGEBRA
INTERMEDIATE ALGEBRA
PLANE GEOMETRY
TRIGONOMETRY
SOLID GEOMETRY
STATISTICS USED IN PSYCHOLOGY, EDUCATION SOCIAL SCIENCE AND ECONOMICS
FINANCIAL AND INSURANCE MATHEMATICS
CHEMISTRY
PHYSICS
COLLEGE ALGEBRA TRIGONOMETRY ANALYTIC GEOMETRY CALCULUS
HIGHER MATHEMATICS WITH APPLICATIONS IN PHYSICS, CHEMISTRY, ENGINEERING, STATISTICS
CAREERS IN INVESTMENT, INSURANCE AND MANAGEMENT
MEDICINE, DENTISTRY, PHARMACY LABORATORY TECHNICIAN
FORESTRY, CONSERVATION, VETERINARY MEDICINE, AGRICULTURE
TEACHING HIGH SCHOOL MATHEMATICS AND SCIENCE
ENGINEERING
SCIENTIFIC RESEARCH IN INDUSTRIAL DESIGN AND DEVELOPMENT
→ MEANS NECESSARY
- - → MEANS DESIRABLE
FOR DETAILS, CONSULT THE CATALOG OF THE COLLEGE TO WHICH YOU INTEND TO GO.

PROVIDING GUIDANCE

The more teachers concern themselves with helping pupils to be competent in the many problems of their daily living, the more these teachers come to realize that guidance is indispensable in every classroom today. Indeed, it has become an educational truism that in educating for citizenship the curriculum of every subject should be guidance-centered, and every teacher should be a counselor.

A book could be written on the opportunities for guidance by the teacher of mathematics. This bulletin has attempted to incorporate certain features in the various parts of the outline and in the various courses. The previous section on "Articulation" shows how teachers may help pupils see the necessary relationships in their school and out-of-school experiences. Personal difficulties in school often come from lack of understanding, from poor study habits. Definite suggestions on improving study in mathematics may be found in this chapter.

A big problem centers around the matter of placement. Placement is a two-fold matter: (1) where to place the pupil and (2) where to place the mathematics. By considering the pupil first and becoming familiar with his abilities, his needs, and his purposes, the value of curriculum guidance becomes of greatest importance. The courses in Chapter III are predicated upon attention to the pupil first. A discussion of guidance for an appropriate program as outlined in Chapter III may be found in the section describing the "Common Learnings Program."

The intentions of the pupil must also be met by a program of studies in mathematics. These intentions are sometimes vocational, sometimes recreational. A valuable service to the pupil may be made by proper personal direction into a vocation which will satisfy his wishes and suit his abilities. The "Guidance Pamphlet in Mathematics for High School Students," the final report of the Commission on Post-War Plans of the National Council of Teachers of Mathematics, November 1947, is a "must" in every mathematics classroom.

The major emphasis for the mathematics program in the tenth year is upon the earner, how mathematics can help the pupil in his choice of and preparation for occupations. The whole problem is graphically summarized by the sketch on "How High School Mathematics Can Contribute to Your Career," and by the charts in Appendix 2, page 283, showing some available occupations which involve the use of mathematics.

The teacher of mathematics is growing more aware of the fact that he is not a taskmaster of mass instruction but is a personal guide for a group of young students who have many personal and learning problems which the teacher and the pupil can attack together.

CHAPTER III

PROGRAM OF STUDIES: SCOPE AND SEQUENCE

A MASTER PLAN

In suggesting certain courses for this bulletin, the planning committee accepted certain basic assumptions. It is important that the teacher be aware of these conceptions so that the subsequent recommendations can be viewed with an understanding of the place of mathematics in the total educational program of the school.

1. Basis

The educational program of any public secondary school should be based upon the needs of pupils, both as present learners and as adults later. It is believed that mathematics has a definite contribution to make toward meeting many of these needs. Mathematics presents an opportunity to develop common understandings in matters pertaining to thought processes as well as to concepts of time and space. As such, appropriate aspects of mathematics should be a part of the education of every youth in the secondary school.

2. Continuing Program

Much of the ineffectiveness of mathematical teaching for use in later life can be traced to two basic causes— (a) lack of meaning to the student, (b) lack of continued contact and use. This manual attempts to underscore the drive for putting meaning into the mathematics program. The second point, concerning continued development, calls for a program of mathematics not just *in* the secondary school but *throughout* the secondary school. Entirely too many young people cease studying mathematics at the end of the ninth or tenth school year. By the time they take their place in society as young adults—two or three years later—they have forgotten much of their mathematics instruction simply because they have not put it to systematic use.

3. Maturation

We learn new things by associating them with things already known, and we gain greater understanding as the uses and needs of mathematics become more urgent and real. Thus, the topic of insurance is more meaningful to a boy about to get a driver's license than it is to a boy in the eighth grade. Consequently, a mathematics program should be developed that will be not only continuous in building up an interpretative background by giving repeated contacts but will be broadened in scope, or will be intensified, so as to capitalize upon the maturation of the learner.

4. Individualization

Learning is an individual process. The offerings of mathematics should give recognition to individual differences and needs. Much of the individualization can be recognized and cared for in the classroom, but a "Master Plan," such as is being presented in this section, must reflect differences. Fortunately for the curriculum planner, human beings are more alike than different. This fact allows us to build a program of "common learnings" which will be of value to all individuals in the modern social order.

All youth need to have experience with the social values of mathematics and with directed practice in the process of thinking especially in the "if-then" relationship involved in daily reasoning. Whatever the future educational and vocational plans of each pupil may be, it is reasonable to think of him as a citizen, as an earner, as a consumer, and as a user of leisure time. Thus, there is ample common ground for a program in mathematical training and understanding for all high school pupils.

It must be remembered, however, that even in a "common learnings" program each person will approach the problems in a different way and will develop different understandings and behaviors. Such a common learnings program needs to be built on a wide variety of learning activities. Therefore, this study guide outlines several courses in mathematics aimed at preparing pupils for certain types of advanced study and for certain types of vocations.

In one group we find those who are especially interested in the fields of science, engineering, pure mathematics, mathematics teaching, and various kinds of research and technical work.

In the other group, there will be those who are not particularly interested in the science of mathematics but in the use of mathematics as it can assist them in their vocational endeavors. They will want to be well prepared in mathematics to the extent that they can interpret mathematical statements pertaining to their life's work. They will want to be prepared to understand and to perform the mathematical processes demanded of the worker in the modern shop, plant, factory, farm, garage, home, store, and office.

Still others will need remedial work suggested by deficiencies in their own work programs or in other life experiences.

5. Conclusion

An analysis of the preceding assumptions leads to the conclusions that an adequate educational program in the modern secondary school should:

a. Include instruction in mathematics for all pupils
b. Offer a continuing program in mathematics
c. Consist of common learnings in mathematics for all plus special educational opportunities for some

Mathematics Program for the Secondary School

The following program is suggested to help meet these general and special needs in the area of mathematics:

Grade	Common Learnings[1]	Special Education[2]
7	Mathematics 7	
8	Mathematics 8	
9	Basic Mathematics I (emphasis upon the individual)	Algebra I
10	Basic Mathematics II (emphasis upon the earner)	Algebra II
11	Basic Mathematics III (emphasis upon the citizen)	Geometry (plane and selected elements of solid)
12	Basic Mathematics IV (emphasis upon the consumer)	Specialized Mathematics a. Plane Trigonometry—1 semester b. Consumer Mathematics—the equivalent of one period a week throughout the year c. Topics selected as to need from the following: Solid Geometry Advanced Algebra Elementary Statistics Analytic Geometry Mathematics Review

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[1] Business mathematics should be developed from these courses in accordance with the specialized needs of the pupils interested in such vocations.

While this may be a full-time program for those pupils without clearly defined vocational and educational plans for the future, it should be recognized that remedial mathematics may serve the needs of some individuals in this group, and for them remedial work should constitute their special program.

Recommendations for work in related mathematics are included in Bulletin 330, *Vocational and Industrial Education in Pennsylvania.*

[2] Some of the important elements of social values in mathematics should be incorporated into the special education courses—taking as much as one-fourth to two-fifths of the time where needed.

MATHEMATICS IN GRADES SEVEN AND EIGHT

The mathematics in grades seven and eight should be closely coordinated with the arithmetic program of the elementary grades as outlined in Bulletin 233-B, *The Elementary Course of Study*. It is essential that the teacher of mathematics in grades seven and eight be thoroughly familiar with the philosophy of Bulletin 233-B and with the content material suggested, especially pages 297 to 339. Frequent checks should be made so that there will be no noticeable or decided gap between the work of the elementary grades and the junior high school.

In the mathematics program of these two years the pupil should make a substantial beginning towards achieving functional competence in mathematics. The specific objectives outlined as necessary for functional competence have been set forth in the form of a Check List by the Commission on Post-War Plans of the National Council of Teachers of Mathematics. The Check List is reproduced on page 101. Naturally the mathematics program of these two years should also provide a dependable foundation for subsequent courses in mathematics. This means that although the objectives are aimed at the pupil's personal use in his current life activities, they also have regard for his future needs as an adult citizen.

The content material of the mathematics program in grades seven and eight is largely arithmetic work, with some work in informal geometry and in elementary concepts of algebra. There are many ways of organizing and presenting the material, and the suggestions given in this guide are not to be considered as rigid requirements. They may be modified to meet the needs of slow-learning children and expanded to challenge children with exceptional ability. In essence this should be considered a two-year "block" of time of both continuing and expanding understandings in mathematics. Thus, grade level sequence should be minimized except to the extent of difference that a year makes in pupil maturation.

Whatever the learner's ability, the teacher should strive to make the work meaningful. Grade placement of the content material is planned to meet the needs of the average school and must also be adapted to special needs. Where there is repetition, the emphasis should be on more extensive application to life situations.

From the standpoint of arithmetic, the work of these two grades is comprised of percentage, mensuration, and such increased proficiency in the fundamental operations with integers, fractions, and decimals as is needed to handle successfully the problems of personal business and vocational life. Although there are not so many new concepts to be learned in these grades, there is need for a definite maintenance program. Concepts and facts learned in previous grades are easily forgotten unless there is systematic review which is aimed, not at meaningless "drill," but at increasing understanding.

Every week all children should do some work which involves the concepts and processes previously learned, and they should realize that practice for speed and accuracy is just as necessary in this field as in athletics or in any other field. There is also great need to develop through continued experience arithmetical work on a higher and higher level. Note the thought expressed in Bulletin 233-B, page 297: *Arithmetic is more than a set of specific skills and facts. It is a system of quantitative and qualitative thinking, including facts, concepts, principles, and processes. . . .*

From the standpoint of informal geometry, the objectives are those of learning the important geometric facts of everyday life. This informal geometry is of a practical experimental type. It has been characterized by the following adjectives: concrete, observational, inventional, intuitional, mensurational, experimental, constructive, and factual. In informal geometry, scale drawing, measuring, counting, paper folding, cutting, matching, and using squared paper are encouraged. The instruments that may be used are unlimited. Common among them are rulers, compasses, protractors, T-squares, drawing paper, cross-section paper, construction paper, tapes, scissors, glue, strings, thumb tacks and stakes.

Since algebra is somewhat more abstract than the informal geometry, it is suggested that it be given preference in grade eight because of the increased maturity of pupils. The work in algebra should be a natural outgrowth of arithmetic. Algebra, as extended generalized arithmetic, should be begun as a shorthand type of arithmetic. Ample practice should be provided for placing ordinary written or spoken sentences into the shorthand language of algebra and vice versa. Work with graphs leads easily into the idea of directed numbers. The idea of having letters represent general quantities or concepts cannot be stressed too much. It is particularly necessary to have beginning work in algebra assume a practical or useful appeal to the student by applying it to real problems.

Just as Bulletin 233-B urges that instruction in any particular subject area should not be limited to the class period in that subject, so also the mathematics program of grades seven and eight should be viewed from two angles: (1) the program of the regular mathematics class and (2) the emphasis that can be placed on all mathematical aspects of experiences encountered in the classes of other school subjects or in general school activities. This requires close cooperation among the teachers of all school subjects. The mathematics teacher should put forth special effort to provide practice that will help the pupil become intelligent and efficient in dealing with the problems that he may meet in other school subjects, in the home, and in his everyday reading and conversation. Similarly, the teacher of every subject should be alert in recognizing and analyzing the mathematical issues arising in his teaching. These cooperative efforts aimed at a continuous study of the mathematical aspects of children's experiences as they arise in the course of daily living can be exceptionally helpful in establishing mathematical meanings and understandings.

Children entering junior high school from different elementary schools may have been taught to perform certain processes by different methods. The important thing in junior high school is not that they must all use the same method but that they understand what they are doing. If they do understand, no effort should be made to have them change their methods. If they do not understand, reteaching and review should be done in a way that will not confuse them.

Purposes of Mathematics in Grades VII and VIII

The following objectives of mathematics for grades seven and eight are goals for pupil development, and, therefore, are guides for teachers in helping pupils to:

1. Interpret quantitative experiences intelligently
2. Develop and maintain behaviors of accuracy and precision
3. Become conscious of the contributions of mathematics to society
4. Recognize and appreciate mathematics in their everyday life
5. Develop a logical and independent approach to the solution of quantitative situations
6. Develop concepts and vocabulary basic to quantitative thinking and the understanding of the decimal system of number notation
7. Attain proficiency and develop the interest necessary for continued use and study of mathematics
8. Develop personal and social behaviors that will contribute to good character growth
9. Discover through mathematics opportunities for the use of leisure time

Areas of Activity in Seventh and Eighth Grade Mathematics

GRADE SEVEN

I. NUMBER SYSTEM

A. *Develop an appreciation of the importance of our number system by use in meaningful situations.*

Meaningful situations may be provided through problems dealing with time, money, age, weight, distance, and speed when these are made realistic to the pupils.

B. *Read and write numbers through billions.*

1. Commas should be used and correctly placed to facilitate reading and writing of large numbers.
2. The word "and" should not be used when reading whole numbers; it should be used only when reading the decimal point and between the whole number and fraction.
3. Give practice in reading and writing large numbers having social significance as found in daily newspapers and magazines—election returns, community chest goals, taxes, etc.

C. *Develop ability to round off numbers.*

1. Whole numbers to tens, hundreds, thousands, and millions.
2. Decimals to tenths, hundredths, and thousandths.

D. *Review Roman numerals with emphasis on today's uses.*

E. *Use historical materials.*

Discuss early devices for counting, such as pebbles, abacus, notches on sticks. Pupils are particularly interested in lattice methods, "sluggard's rule," etc.[1] This historical approach to an old or new topic will help to increase pupil interest and understanding.

II. FUNDAMENTAL OPERATIONS

A. *Review fundamental processes with integers, common fractions, and decimal fractions.*

Use standardized diagnostic tests and provide for necessary reteaching.

B. *Whole numbers*

1. Limit addition of large numbers to five addends.
2. Limit subtraction to numbers of six digits.
3. Limit multiplication generally to multipliers of three digits.
4. Extend division to divisors of three or four digits.

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[1] See such teacher references as
B. R. Brokingham, *Elementary Arithmetic: Its Meaning and Practice* (Ginn).
H. D. Larsen, *Arithmetic for Colleges* (Macmillan).

5. Teach short form of division with divisor of one digit, with and without remainder:

$$\begin{array}{r} 85 \\ 9\overline{)765} \end{array} \qquad \begin{array}{r} 769\frac{2}{5} \\ 5\overline{)3847\phantom{\frac{2}{5}}} \end{array} \qquad \begin{array}{r} 1256\frac{2}{3} \\ 6\overline{)7540\phantom{\frac{2}{3}}} \end{array}$$

6. Teach various ways of expressing remainders.
   a. As a fraction in lowest terms
   b. With a + plus sign

C. *Fractions*

1. Change fractions to decimals, exact or to the nearest tenth, hundredth, or thousandth.

$$\frac{7}{8} = \begin{array}{r} .875 \\ 8\overline{)7.000} \end{array} \qquad \frac{5}{9} = \begin{array}{r} .555\frac{5}{9} \\ 9\overline{)5.000\phantom{\frac{5}{9}}} \end{array} \text{ or } .556$$

2. Problems involving money—getting answers to the nearest cent

$$\frac{2}{3} \times \frac{\$.25}{1} = \frac{\$.50}{3} = \$.16\frac{2}{3} \text{ or } \$.17$$

D. *Decimals*

1. Addition and subtraction of "ragged decimals"
   7.86 + 39.7 + 86 + 19.763 = 153.323
   17 - 8.417 = 8.583
   13.6 - 9.84 = 3.76
   26.832 - 19 = 7.832

2. Division of a smaller number by a larger one, answers exact or to nearest tenth, hundredth, or thousandth, or to nearest cent

$$\begin{array}{r} .72 \\ 25\overline{)18.00} \\ 17\ 5 \\ \hline 50 \\ 50 \\ \hline \end{array} \qquad \begin{array}{r} \$0.38\frac{1}{6} \\ 18\overline{)\$6.87\phantom{\frac{1}{6}}} \\ 5\ 4\phantom{7\frac{1}{6}} \\ \hline 1\ 47\phantom{\frac{1}{6}} \\ 1\ 44\phantom{\frac{1}{6}} \\ \hline 3\phantom{\frac{1}{6}} \end{array} \text{ or } \$0.38$$

3. Multiplying and dividing by 10, 100, or 1000 by moving the decimal point

E. *Problem-solving*

1. Make problem-solving an important part of the year's work. Use problems that are meaningful and of local interest.

2. Develop ability to read problems carefully; to make an estimate of the answer; to draw a sketch when possible; to reject irrelevant data; to analyze relations; to judge the reasonableness of the answer; and finally to check for accuracy.

III. PERCENTAGE

Pupils already know decimals and some fractions; so this is a good opportunity to emphasize the relationship of per cent to common fractions and decimal fractions, to show that the only new thing in percentage is a new symbol and a new way to express an idea. Develop the concept that this is just one of the three common ways of expressing a part or a ratio.

A. *Discuss informally the use of per cent in everyday life.*

B. *Teach meaning of per cent as hundredths, or as another way to express a fractional part.*

$7\% = .07 \qquad 23\% = .23 \qquad 100\% = 1.00 \qquad 2\frac{1}{2}\% = 2.5\% = .025$

C. *Use per cents from 1% to 100%.*

D. *Teach how to change per cents to decimals and decimals to per cents by moving the decimal point.*

E. *Teach how to change fractions to per cents by changing first to a decimal and then to a per cent.*

$$\frac{13}{20} = \begin{array}{r} .65 \\ 20\overline{)\,13.00} \end{array} = 65\%$$

F. *Explain how to change per cents to fractions.*

$$25\% = \frac{25}{100} = \frac{1}{4} \qquad 12\tfrac{1}{2}\% = .12\tfrac{1}{2} = 12\tfrac{1}{2} \div 100 = \frac{\cancel{25}}{2} \times \frac{1}{\underset{4}{\cancel{100}}} = \frac{1}{8}$$

The following fractional equivalents should be introduced through meaningful situations and then practiced until the equivalents become synonymous:

$20\% = \frac{1}{5}$
$25\% = \frac{1}{4}$
$37\frac{1}{2}\% = \frac{3}{8}$
$66\frac{2}{3}\% = \frac{2}{3}$
$50\% = \frac{1}{2}$
$12\frac{1}{2}\% = \frac{1}{8}$
$33\frac{1}{3}\% = \frac{1}{3}$
$80\% = \frac{4}{5}$
$62\frac{1}{2}\% = \frac{5}{8}$
$16\frac{2}{3}\% = \frac{1}{6}$
$60\% = \frac{3}{5}$
$10\% = \frac{1}{10}$
$83\frac{1}{3}\% = \frac{5}{6}$
$40\% = \frac{2}{5}$
$75\% = \frac{3}{4}$
$87\frac{1}{2}\% = \frac{7}{8}$

G. *Teach how to find a given per cent of a given number* (Type I).

Consider a problem such as: John was promised 15% of all the money he took in for selling lemonade. What was his share of a day's sales totaling $4.20?

1. Use decimal equivalents to show that "per cent of" means "hundredths times."
   18% of $95 = .18 \times 95 = 17.10$
   100% of $67 = 1 \times 67 = 67$
2. Use fractional equivalents.
   25% of $24 = \frac{1}{4} \times 24 = 6$
   $16\frac{2}{3}$ of $36 = \frac{1}{6} \times 36 = 6$
   (Note: Limit per cents to whole numbers except for those fractional equivalents listed above.)

H. *Teach how to find what per cent a given number is of another given number* (Type II)

The class has agreed to try to raise $12.00 for the Cancer Fund. They now have $8.00. What per cent of their goal has been reached?

1. Find what fractional part one number is of another, change the fraction to a decimal, and then to a per cent.
   What % is 19 of 25?

$$\frac{19}{25} = 25\overline{)19.00}^{\;.76} = 76\%$$

2. —— or in the case of equivalents
   18 is what % of 27?

$$\frac{18}{27} = \frac{2}{3} = 66\frac{2}{3}\%$$

IV. COMMON USES OF PERCENTAGE IN EVERYDAY LIFE: Discount, commission, profit and loss, interest, increase and decrease.

A. *Percentage may be presented as a family.* Like any family, there are certain things common to the whole family. Tell the pupils that they are going to meet several members of the family and that the family characteristics will be found in the children. The "children" are:

Interest *Discount* *Commission*
Profit and Loss *Increase and Decrease*

B. *Discount*

1. Select a situation encountered by a consumer—such as "discount for cash," "discount to employes"—and through discussion develop the meaning of the following terms:
   Discount
   Marked price, regular price, or list price
   Net price or selling price
2. Explain that the net price is found by subtracting discount from regular price:
   Net price = Market price – discount

Optional:

Net price may be found by first subtracting rate of discount from 100%, getting rate of net price. Then,

Net price = rate of net price × regular price.

C. *Commission*

1. Discover pupil experiences with commission—such as selling newspapers, magazines, greeting cards, part of father's income—and formulate definitions in pupil's own language of:
    Commission
    Rate of commission
    Net proceeds
    Sales
2. Relate to percentage by showing that the sales are the base, or b; the rate of commission is the rate, or r; and the commission is the percentage, or p. Then,
    Commission = Rate × Sales
    $$\text{Rate of commission} = \frac{\text{Commission}}{\text{Sales}}$$
3. Net proceeds equal sales minus commission.
    (Optional method may be used in finding net proceeds just as in finding net price.)
4. Commission on selling papers and magazines.

D. *Interest*

Approach the problem through a discussion and study of situations encountered in pupils' families, such as loans from banks or finance companies, installment buying, lay-away plans.

In the discussion, introduce and develop definitions of:

Interest
Principal
Rate of interest
Time

Explain the added feature of time and that time must always b expressed in years.

Rate × principal = interest (one year)

If time is not one year, there must be a further step—multiplying by the time. Hence the formula:

Interest = Principal × Rate × Time (years)

Examples:

a. Interest on $500 at 4% for two years would be:
   Interest = $500 × .04 × 2
b. Interest on $300 at 4% for 8 months would be:
   Interest = $300 × .04 × ⅔

E. *Profit and Loss*

1. Explain the meaning of the following terms:

   Cost — Loss
   Profit — Selling price

2. Relate to percentage but explain carefully that profit or loss may be based either on cost or selling price.
3. Teach the following formulas:

   Cost + profit = selling price
   Cost – loss = selling price

4. If profit or loss is based on cost,

   Profit or loss = rate × cost

   $$\text{Rate of profit or loss} = \frac{\text{Profit or loss}}{\text{Cost}}$$

5. If profit or loss is based on selling price,

   Profit or loss = rate × selling price

   $$\text{Rate of profit or loss} = \frac{\text{Profit or loss}}{\text{Selling price}}$$

F. *Increase or Decrease*

1. Explain meaning of increase and decrease.
2. Relate to percentage.

   Examples:

   a. Sales first week were $150.
      Sales second week showed 30% increase.
      Increase = 30% × $150
   b. Attendance first day was 40.
      Attendance second day was 38.

   $$\text{Rate of decrease} = \frac{2}{40} = .05 = 5\%$$

G. *Summary Activity:*

Have pupils bring in newspaper and magazine articles or advertisements in which per cents, fractions, and decimals appear. Sort these into appropriate folders. Pupils then select items from a folder and rewrite the key phrases, using the other two types of expression; i.e., if the story or advertisement uses a fraction, retell it—(a) using a decimal, (b) using a per cent. Discuss why the part containing the key phrase in the original article or advertisement was or was not more effective.

V. INFORMAL GEOMETRY

A. *Develop ability to recognize and sketch the basic geometric forms of everyday life,* such as:

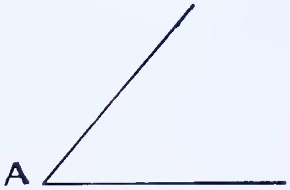
Square
Circle
Pyramid
Rectangle
Cube
Rectangular Solid
Sphere
Parallelogram
Triangle
Trapezoid
Cone
Cylinder

1. Have pupils find illustrations of geometric forms in classroom and community.
2. List geometric forms found in nature—fruits, shells, snowflakes, spider webs, etc.
3. Use pictures of buildings and have pupils identify the geometric forms found in them.
4. Discuss the shapes of boxes, cans, etc.
5. Teach the origin of the word *geometry*—"geo," meaning *earth*, and "metric," meaning *measure*
6. Use geometric figures in designs.

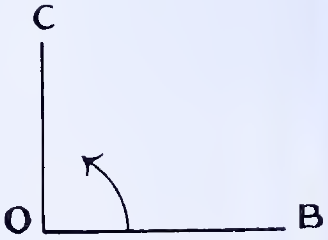
B. *Develop ability to use ruler, compass, and protractor as geometric tools.*

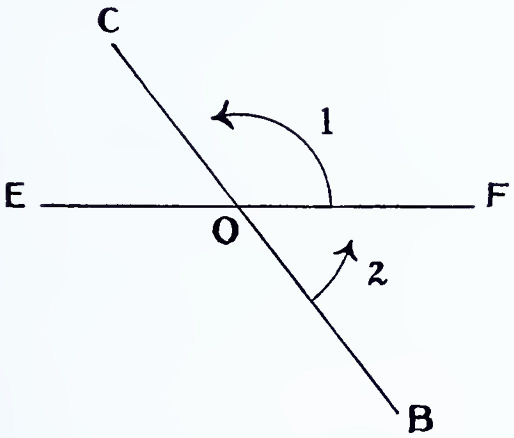
1. Lines
    a. Draw and identify the following kinds of lines:
        Straight Curved Broken Parallel
        Intersecting Perpendicular Horizontal
        Vertical Oblique
    b. Have pupils find illustrations of various kinds of lines in the classroom and in the community.
    c. Practice drawing and identifying line segments as to their position and relation to other lines.
    d. Show how to erect perpendiculars.
2. Angles
    a. Use blackboard compass, scissors, and hands of clock to develop the concept of angles.
    b. Teach meaning of the following kinds of angles:
        Acute Obtuse Right Straight
    c. Teach how to label and read angles.

Read: "Angle A"



Read: "Angle BOC" or "Angle COB"





Read: "Angle 1" or "Angle FOC" or "Angle COF"

Read: "Angle 2" or "Angle BOF" or "Angle FOB"

- d. Explain the meaning of degrees and show how they are used to measure angles.
- e. Practice drawing angles of various sizes and measuring them with a protractor.
- f. Show how to bisect lines and angles with a compass.

3. Arcs and circles
    - a. Discuss contribution of the wheel (circle) to society.
    - b. Show use of circle and arcs in art, architecture, etc.
4. Use all tools in construction of simple geometric figures, such as squares, rectangles, triangles, parallelograms, and hexagons.

VI. MEASUREMENT

A. *Review, reteach, if necessary, the work in denominate numbers of previous grades.*

B. *A brief history of measurements would be interesting to children in this grade.*

C. *In reviewing linear measures, do a good bit of practice work in estimating answers. Show that the degree of accuracy in measuring depends on what is measured, the reason for measuring, and the instruments used.*

D. *Be sure pupils understand the divisions on a ruler to sixteenths of an inch.*

E. *Perimeters and circumference*

1. Explain the meaning of perimeter as the measurement around the edge of a plane figure.

2. After measuring the perimeters of a number of figures, develop formulas for finding perimeters of the following figures:
    - (a) Square $P = 4s$
    - (b) Rectangle $P = 2(l + w)$ or $P = 2l + 2w$
    - (c) Parallelogram $P = s + s + s + s$
    - (d) Triangle $P = s + s + s$
3. Show that all dimensions must be measured in the same unit.
4. Provide practice in changing from one unit to another.
5. Teach meaning of radius, diameter, and circumference of the circle.
6. Measure a number of circles by rolling circular objects along a yard stick. Measure the diameters of the circles and in each case note the quotient obtained when the circumference is divided by the diameter. Each quotient should be slightly more than 3 which gives rise to the concept of "pi," π.
7. Teach the following formulas:

    $D = 2r$

    $C = \pi d$ or $C = 2\pi r$

F. *Areas*

1. Teach the meaning of area as the measure of the surface of a plane figure.
2. Explain that linear measure cannot measure surface of area.
3. Explain the meaning of square measure, putting 2 square inches, a square foot, and a square yard on the board.
4. Develop the table of square measure.
5. Have pupils measure squares and rectangles with square inches to find areas.
6. Develop the following formulas where A represents area.

    A (square) $= s \times s$

    A (rectangle) $= l \times w$
7. Explain the meaning of height or altitude of a triangle or parallelogram as the perpendicular distance from the base to the point or side opposite the base.
8. Develop the following formulas:

A (triangle) $= \frac{1}{2} \times b \times h$ *or* A (triangle) $= \frac{1}{2}bh$

A (parallelogram) $= b \times h$ *or* A (parallelogram) $= bh$

A (circle) $= \pi \times r \times r$ *or* A (circle) $= \pi r^2$

VII. GRAPHS

A. Develop an appreciation of the value of graphs for picturing numerical data through reading and interpreting pictographs, line, bar, and circle graphs.

B. Have pupils bring graphs from magazines, newspapers, science and social studies textbooks, etc.

C. Teach how to construct line and bar graphs—simple ones with no interpolations or very easy ones.

D. Select data for graphs which are meaningful to pupils, such as attendance records, temperatures, P.T.A. memberships, football and basketball games, etc.

ILLUSTRATIVE UNIT

DISCOUNT

1. *Overview by Teacher and Planning with Pupils*

2. *Central Objective*

To develop an understanding of the use of discount in everyday life.

3. *Teacher Planning*

a. Choose a suitable time to teach the unit. (After Christmas vacation is a very appropriate time. The sales advertised in the local papers provide useful material.)

b. Have class discussion to learn what knowledge the pupil has concerning the unit.

c. Lead pupils to tell of experiences they have had in buying at reduced prices.

d. Develop reasons why the pupil or his family bought merchandise when there were sales.

e. Show newspaper advertisements of merchandise for sale at reduced prices.

f. Use pictures that illustrate discount. (Most textbooks have good illustrative material for this unit.)

g. Arouse interest by pertinent questions (have children make and ask questions) such as:

(1) Why did Mary wait till after Christmas to buy her new coat?

(2) Why did your mother say that she must pay her electric bill by the twelfth of the month?

(3) Why did your parents buy a case of canned peaches?

(4) Why did John's father buy a bicycle for him at the place where he is employed instead of at another place in town?

(5) If soap is selling at 2 cakes for 21¢, why would it be better to buy two cakes at once than to buy one cake today and another several days later?

(6) Is it possible to buy something at a discount that would really not be a saving?

h. Correlate discount and thrift.

4. *Orientation*

Students of today will be the people who carry on business and manage homes in future years. They will need to understand material that refers to business and homemaking. This unit will provide a real purpose for understanding the per cent unit that has just been completed, and will show how percentage is used in the business world. It is natural to be alert for bargains and to be thrifty buyers. Discount deals with thriftiness in such matters.

Suggested situations:
Buying for a new home.
Buying equipment for school sports program—read advertisements
What is typical educational discount?

a. Have class discussion concerning:
   (1) Bargains and how advertised
   (2) Motives of stores when they conduct sales
   (3) Kinds of sales

b. Provide some good examples of discount through advertising.

c. Establish the idea that this unit fits in with these advertisements.

d. Read problems from text and make comparisons with real advertisements of local stores.

e. Provide thought-provoking questions pertaining to material that is known and can be used to find discount and prices after discount is deducted.

f. Use some drill work to review these methods.

g. Use a real advertisement as a basis for a problem and develop the learning process necessary for solving.

Example:

This advertisement said Brown's are selling boys' $6.50 sweaters at 25% off.

(1) What does this mean?
(2) The 25% will be off what amount?
(3) How could you find that?
(4) What would you save? Is that amount worth saving?
(5) What would you pay for the sweater?

h. Find a similar problem in the texts.

i. Guide class discussion in the value of this unit and its usefulness.

j. Have class evaluation of understanding per cent in order to understand discount.

5. *Learning Period*

a. Have pupils bring to class different kinds of newspaper advertisements that illustrate discount.

b. Have pupils bring bills showing discounts.

c. Read, study, and discuss these.

d. Make a list of different ways discount is expressed.

e. Committee work on preparing problems for class work from newspaper advertising.

f. Individual problem making.

g. Encourage working together and comparing work with others for correctness, accuracy, and method.

h. Consult parents and have them help estimate monthly and yearly savings by making use of discounts.

i. Have committees consult school purchasing personnel to find out how they save by discounts.

j. List sales from local papers that affect the pupil, his parents, or the family.

k. Make a list of words common in discount and use them.

l. Solve problems keeping in mind that they are real problems involved in living.

6. *Culminating Activity*

Plan a sale.

a. Bring personal possessions that might be offered for sale.

- b. Have committee workers as: managers, salespeople, those in charge of displaying merchandise, buyers (satisfied and dissatisfied), complaint department, etc.
- c. Give a second discount on articles not selling well.
- d. Pupil committees work with the teacher and prepare a test from this sale that will test the comprehension of the unit.

7. *Evaluation* (by teacher)
    - a. Are pupils more conscious of the need of arithmetic in their everyday living when it involves a real experience?
    - b. What words were acquired through their use in the unit?
    - c. To what extent did pupils make use of newspapers to learn about sales?
    - d. How much has class participation been improved?
    - e. Are the pupils able to discuss intelligently the value of discount and to understand how it affects their living?

GRADE EIGHT

I. ARITHMETIC

A. *Whole numbers, fractions, and decimals*

1. Use inventory tests to discover weakness in fundamental processes and provide remedial work as needed.
2. Maintain facility in fundamental processes through meaningful drill at regular intervals.
3. Review the following short methods (emphasize *understanding):*
    - Multiplying and dividing numbers by 10, 100, 1000
    - Fractional equivalents for certain per cents
4. Use numbers in items in the newspapers for practice in reading and comprehending numbers.
5. Teach the meaning and method of finding averages.

    The following types of data may be collected and used in finding averages:

    - Weather reports
    - Size of classes in school
    - Daily attendance records
    - Test scores (if all are made on same basis)

B. *Graphs*

1. Review work of previous grades.

2. Practice in reading and constructing more difficult line and bar graphs where interpolations are needed.
3. Teach how to construct simple circle graphs.

C. *Percentage*

1. Review per cents as taught in seventh grade, particularly
    - Meaning of *hundredths*
    - Fractional and decimal equivalents
    - Finding a given per cent of a given number
    - Finding what per cent one number is of another
2. Teach the meaning and use of per cents less than 1% and over 100%.
    a. Give practice in writing common fractions as decimals:
    ½ = .5   ¾ = .75   ⅝ = .625

    b. Contrast a fraction with a fractional % and show that they are not the same
    ½% is ½ of 1% and not ½
    ¼% is ¼ of 1% and not ¼

    c. Give practice in changing per cents to decimals:
    ½% = .00½ = .5% = .005
    ¾% = .00¾ = .75% = .0075

    d. Show the various methods of finding ½% of any number:
    Find 1% of the number and take ½ of the result.
    Change ½% to .00½ and multiply.
    Change ½% to .005 and multiply.

    e. Show pupils that if 100% = 1, then any per cent over 100% must be more than 1:
    118% = 1.18   300% = 3

    f. Provide practice in moving the decimal point to change a per cent to a decimal and a decimal to a per cent.
3. Review percentage problems of Types I and II, using per cents less than 1% and more than 100%. (See G and H under Grade Seven, pages 76 and 77.)
    Type I—Find a given % of a given number
    ½% of $500 = .005 × $500
    116% of $400 = 1.16 × $400
    Type II—What per cent of a given number is a given number?
    What % of $400 is $2?
    What % is $70 of $35?

4. Teach how to find a number when a per cent of it is given—Type III

   Introduce such exercises as:

   6 is ½ of ——? 6 is 50% of ——?
   4 is ¼ of ——? 4 is 25% of ——?

Show that basic formula learned in 7th grade, $r \times b = p$, may be used for problems of Type III. (See C, 2, page 78.) Thus:

$$b = \frac{p}{r}$$

5. Give practice in the solution of all types of percentage problems:

   Review again that all three percentage formulas are derived from the original $p = r \times b$, and that all can be solved by that formula.

6. Review the use of percentage in: Commission, simple interest, profit and loss (based on cost or selling price), simple discounts, increase and decrease (pages 77-79).

Apply the skill learned in solving percentage problems of Type III to find:

The principal when interest and rate are given
The regular price when discount and rate of discount are given
The amount of the sale when commission and rate of commission are given
The cost or selling price when profit or loss and rate of profit or loss are given
The base number when increase or decrease and % of increase or decrease are given

D. *Measurements*

Review and reteach, if necessary, the measurements taught in previous grades.
Introduce the metric system.
Simple volumes should be considered.

E. *Intelligent Use of the Family Income*

1. Banking
   a. Discuss the services rendered by a bank.
   b. Show how to open an account—get sample forms from a bank.
   c. Point out differences between a checking and a saving account.

   d. Give practice in filling out deposit slips.
   e. Discuss checks and their uses.
   f. Show pupils how to make out checks, fill in stubs, and the different ways of endorsing checks.
   g. Visit a bank if possible.

2. Insurance

   When do we need insurance? How can we identify a reliable adviser? (We mustn't jeopardize the future by selecting the wrong type of insurance.)

   a. Discuss the meaning and importance of insurance as a cooperative undertaking for mutual protection.
   b. Discuss the most important kinds of insurance:
      Life
      Fire
      Automobile liability
      Insurance on parcel post shipments
      Health and accident
      Group hospitalization
   c. Develop an understanding of the following terms:
      Policy
      Premium
      Rate of Insurance
      Face Value
      Dividend
      Beneficiary
   d. Discuss factors which determine premium rates on different kinds of policies.
   e. Teach how to compute premiums in simple problems when such rates are given as
      \$.35 per \$100
      \$8.75 per \$1000
   f. Pupils may be interested in giving reports on unusual kinds of insurance.
   g. Have an insurance representative visit the class.

3. Budgets

   a. Discuss the need for thrift in the use of time and money and how budgets may help in using time and money most advantageously.
   b. Make simple budgets, both personal and family. Pupils at this age level need to develop and follow time budgets.
   c. Suggestions for making graphs:
      How I spend my allowance
      How I spend my day
      (particularly well adapted to circle graphs)

4. Taxes
   - a. Discuss the necessity for taxation, why we pay taxes, what we receive for them, the services of government. Have pupils make a list of the activities paid for by tax money.
   - b. Point out that taxes could be greatly reduced if people were honest and just in their dealings with each other. Have pupils make a list of government activities paid for by tax money.
   - c. Teach as a matter of information the most common kinds of taxes:
     Tolls
     Sales taxes
     Property taxes
     Income taxes
     Customs or duties
   - d. Distinguish between national, state, and local taxes. Show that everyone pays taxes in one way or another.
   - e. Explain the different ways tax rates are expressed:
     Mills
     Dollars per hundred dollars
     Dollars per thousand dollars
     Cents per dollar
     Per cents
   - f. Compute taxes in simple problems, particularly sales taxes and taxes where rates are expressed as per cents.

F. *Squares and Square Roots*

1. Use the area of a square to explain that the square of a number is the product of a number multiplied by itself and is written 9^2; 9^2 means 9 x 9 or 81, or a square is the product of 2 equal factors.
2. Explain that the square root of a number is one of two equal factors of the number.
3. Explain meaning of word *factor*.
4. Introduce the radical sign ($\sqrt{\quad}$) as a symbol for square root.
5. Give practice in using the tables of squares and square roots. Also include square roots of fractions: $\sqrt{1/4}$.
6. Better students may be taught how to extract square roots to tenths by the process which resembles division.

II. INFORMAL GEOMETRY

A. *Review plane figures using models and drawings.*

B. *Review areas of rectangle, square, parallelogram, and triangle.*

C. *Teach for recognition the following solids:*

Prisms	Cones
Pyramids	Spheres
Cylinders	

Have pupils collect objects, boxes or cans in the above forms.

D. *Teach the meaning of volume as capacity or what a solid holds.* Use inch cubes to find the volume of a box.

E. *Develop the table of cubic measure.* Contrast inch, square inch, cubic inch.

Have models of these units on display to help pupils distinguish between the three kinds of measures.

F. *Teach how to find the volume of* rectangular solids, cubes, pyramids, cylinders, and cones.

1. Emphasize that all measures must be in the same unit.
2. Build a rectangular solid with 1-inch cubes, and develop rule for finding the volume. Write the rule as a formula: $V = lwh$. [Volume = length × width × height]
3. Treat the cube as a special case of the rectangular solid where length, width, and height are all the same—the length of any edge or side.
   Develop the formula: $V = s^3$; usually $V = e^3$ is used.
4. Use a pyramid and a rectangular solid of equal base and height to show that the volume of a pyramid is one-third of the volume of the rectangular solid, and develop the formula: $V = \frac{1}{3} Bh$, making certain the pupils understand that "B" is the *area* of the base.
5. Show that the number of cubic units in the first layer of a cylinder is the same as the number of square units in the area of the base (circle) and develop the formula: $V = \frac{1}{3}\pi r^2 h$.
6. Illustrate that the contents of three cones of the same height and base as the cylinder will be needed to fill the cylinder. $B = \pi r^2$. Thus the formula for the volume of a cone is: $V = \frac{1}{3} \pi r^2 h$.
7. Point out the need for finding volumes in making both accurate and estimated computations of the relative capacities of different-shaped containers. Example:

   A rectangular solid half as long, half as wide, and half as high as another will contain only one-eighth as much as the larger solid.

8. Keep problems in finding volumes simple and within the experience of the pupils. Experiment with hollow figures that can be filled with salt, sand, or water. For example: If the figures have equivalent bases and altitudes, the cone or pyramid can be emptied three times into the cylinder or prism.

G. *Indirect measurement*

1. Pythagorean Theorem
2. Similar triangles

III. ALGEBRA

A. *Language of algebra*

1. Point out the importance and meaning of symbolism in life through a discussion of signs already known: $, ¢, #, %, @, and abbreviations such as Pa. for Pennsylvania.
2. Explain that not only words but also whole sentences can be stated by symbols. Examples:
   "7 diminished by 5 equals 2" may be written $7 - 5 = 2$.
   "Discount is found by multiplying the rate of discount by the marked price" may be written: Dis. $=$ R $\times$ M.P.
   Use examples of formulas already learned in translating from statement to formula, and vice versa.
   Use statements involving "increased by," "diminished by," "more than," "less than," "exceeds," "divided by," "3 times as much," and translate them into algebraic language.
3. Give much practice in reading and explaining the meaning of algebraic expressions such as:
   $3a$ means 3 times a
   $\frac{a}{2}$ means a divided by 2 or ½ of a
   $2xy$ means 2 times x times y
   $4x$ plus $5y$ plus $8z$ means 4 times x plus 5 times y plus 8 times z. y^2 means y times y. x^3 means x times x times x.
4. Point out that since letters in an algebraic expression stand for numbers, every algebraic expression represents a number. Give plenty of practice in evaluating algebraic expressions using fractional and decimal values as well as integral ones. This type of work offers an excellent opportunity for reviewing fundamental operations.
5. Use formulas from work in other classes, science, shop, home economics, for practice in interpretation and evalua-

tion. Stress the fact that a formula is a rule expressed in symbols.

B. *Equations*

1. The algebraic relationships developed with eighth grade pupils are an outgrowth of quantitative generalizations drawn directly from arithmetic. Thus as a "link" considerable emphasis should be put on an introduction as follows:

$6 + ? = 10$ (Read: "Six plus what number equals ten?")
$? - 5 = 2$
$4 \times ? = 12$
$\frac{?}{7} = 14$

Explain the meaning of *equation* as an expression of equality. This can be developed from arithmetic orally by using "*what number,*" for the quantity later referred to as "*x,*" "*n,*" etc. Have pupils read equations similar to the following and tell what they mean:

$3a = 12$ $y - 7 = 3$

$\frac{x}{9} = 4$ $8 + b = 12$

2. Introduce the expression "solving an equation." Like solving a mystery, it means finding that which is unknown. The letter stands for the unknown. Have pupils solve some easy equations by inspection and check them.

3. In developing the understanding of how to solve equations which cannot readily be solved by inspection, use a pair of scales or a balance. Show that if you do the *same* thing to both sides of the scales they remain balanced. From such illustrations, develop the following axioms:

If equals are divided by equals, the results are equal.
If equals are multiplied by equals, the results are equal.
If equals are added to equals, the results are equal.
If equals are subtracted from equals, the results are equal.

Teachers should realize that these are not self-evident truths but rather are assumptions or laws that mathematicians have agreed shall pertain to algebraic equations.

4. Show how these axioms can be used in solving simple equations. Use only equations involving one step such as:

$4x = 36$ Solve by dividing both sides by 4.

$\frac{y}{7} = 6$ Solve by multiplying both sides by 7.

$x + 3 = 9$ Solve by subtracting 3 from both sides.

$y - 4 = 6$ Solve by adding 4 to both sides.

Do not use equations involving directed numbers. Solve by axioms and meanings, not by transposition.

Through various examples determine what operation to use until the following generalization can be made.

a. Each operation gets rid of some unwanted number.
b. The operation to be used is the opposite of the operation known.

Stress the importance of checking all roots.

5. Before attempting to solve simple problems by the use of equations, give plenty of practice in translating word statements into algebraic expressions such as:

Three times a certain number equals 24: $3x = 24$
The perimeter of a square is 20: $P = 20$ or $4s = 20$

One-third of a number is 11: $\frac{1}{3}x = 11$ or $\frac{x}{3} = 11$

6. Have pupils follows a definite procedure in solving problems such as:

Determine what is to be found.
Let x or any other letter stand for the unknown.
Draw a figure, if possible.
Develop the proper equation by expressing relation between knowns and unknowns.
Solve the equation by using axioms.
Check the answer in the original problem.
Use simple problems involving equations of only one step.

C. *Ratio and proportion*

1. Explain ratio as one of several ways of comparing quantities. It is usually written as a fraction.

There are 20 girls in a class and 40 boys. What is the ratio of girls to boys? $\frac{20}{40}$ or $\frac{1}{2}$

Of girls to total? $\frac{20}{60}$ or $\frac{1}{3}$

Of boys to girls? $\frac{40}{20}$ or 2

Of boys to total? $\frac{40}{60}$ or ⅔

Therefore a ratio may be defined as the quotient of one number divided by another.

2. Explain proportion as a statement of equal ratios.
   $3/4 = 6/8$ may be interpreted as: 3 is to 4 as 6 is to 8 or the relationship of 3 to 4 is the same as the relationship of 6 to 8.
3. Develop an understanding of:
   - Means
   - Extremes
4. Lead pupils to discover the rule of proportion: *The product of the means is equal to the product of the extremes.* Use this rule in finding terms in a proportion.

D. *Numbers with signs* (Optional)

1. Explain the need for negative numbers to show concepts of less than zero.
2. Explain that *0* (zero) is a point of reference as well as a reference to quantity. Have pupils name situations where quantities more or less than zero, or above and below zero, are involved, such as:
   - Temperatures on thermometers
   - Game scores
   - In or out of debt
   - North and South latitude
   - East and West longitude

   Bring out the fact that positive and negative numbers are used to represent opposites or direction.
3. How could the financial status of each of the following be expressed?

One who owes nothing and has nothing?	0
One who owes $3 and has $8?	+ $5
One who has $6 and owes $9?	− $3
One who has $5 and gets $7?	+ $12
One who has $16 and owes $8?	+ $8
One who has $4 and owes $4?	0

4. Stress the difference between a signed number and its absolute value.
5. Point out that plus and minus signs have two meanings:
   - To indicate addition or subtraction
   - To indicate the kind or direction of a number

6. Teach how to add simple directed numbers. Add:

+ 6	– 3	+7	– 7
+ 7	– 8	– 4	+ 2
+ 5	– 2	+5	– 9
——	——	——	——
+18	–13	+8	–14

ILLUSTRATIVE UNITS

UNIT I

A MONTH OF PRACTICAL ARITHMETIC

On the first day of a calendar month, the members of a class may be informed that an imaginary $100 has been deposited in a given bank to the credit of each pupil. This $100 plus any money the pupil may earn is to be used to meet all the financial needs for that month.

To carry out the project, each pupil is required to do the following:

1. Keep an accurate account of all receipts and expenses, and balance the same at the end of the month
2. Write a check and stub for each withdrawal from the bank
3. Make deposit slips for any money earned and added to bank account
4. Make a circle graph showing the expenditures under the following headings: room, board, recreation, health, church and charity, savings, and miscellaneous

When the project has been completed, an interesting class period may be spent comparing the graphs and deciding who has been wise and who foolish in the expenditures of money. Pupils are interested in comparing notes on the amount that parents charge for board and room. The fact that some pupils give nothing to church or charity and that others save nothing should lead to discussions concerning the pupils' place in the community and the need for making provision for the future.

UNIT II

INSURANCE

This outline on Insurance is suggested by a class as a good topic for discussion and action. By way of introduction it is suggested that Insurance be classified into four general groups: (1) Fire, (2) Automobile, (3) Life, and (4) Miscellaneous, including old age, group accident, hospitalization, annuities, pensions, social security, and others.

Four committees are then formed and a chairman is elected from each group, and one of the four classes of insurance is chosen by each group.

Each committee decides to keep a group scrapbook on the type of insurance selected. Each committee then visits various insurance companies for information and literature on that type of insurance and scans the newspapers for current news, checks the library for basic information, and looks through leading magazines for relevant articles.

These materials are then compiled in a scrapbook. Each committee stresses originality. The collection may include: (1) written materials gathered from supplementary sources, from interviews with insurance agents, and from current literature; (2) literature published by various insurance firms; (3) graphs; (4) charts; and (5) maps and statistics.

The committee chairman gives a comprehensive report on the committee's findings, after which a discussion is held on each type.

As a follow-up, local insurance agents may visit the class and speak on various phases of insurance. Each agent might be allotted one class period for his remarks and for questions by the students.

Practical insurance problems to be solved by the entire class should then be selected by the committees and by the teacher, from texts, supplementary books, and workbooks. A survey test should follow.

UNIT III

IS BUYING ON CREDIT DESIRABLE?

1. *Preliminary teacher planning*

At the beginning of the year, after a discussion of the course, the pupils handed in a list of suggested topics for consideration for the year. The group then elected a chairman. (This was done according to parliamentary procedure with voting by ballot.) He, in turn, chose a committee to work out the problem from the suggestions of the class.

They chose the topic "Credit." With the aid of the teacher, they worked out the problem stated above and the procedure. The committee felt that because about 90 per cent of the class had some kind of part-time employment, this topic was a very important one. They also felt that the group was a good cross section of the community.

2. *Orientation*

The problem was presented by the chairman to the class as a whole. He briefly explained the term credit, citing examples within their own experience, for example that of the seniors in buying their class rings.

3. *Learning period*

   Everyone read as much as he could about the forms of personal credit. They talked it over at home with their parents. They contacted merchants in town to get their views on personal credit. They formulated ideas from their own experience.

4. *Culminating activity*

   Students reported their findings. Discussions followed on these findings. Debate: *Resolved,* That installment credit should be confined to the purchase of necessities.

5. *Evaluation*

   a. Each student made a budget. This showed what he could afford to do financially.

   b. A subjective test was given.

   c. This problem brought home to many the things they had not quite realized before about maintaining a home. From their discussions, this group of thirty-four are going to stop and think before they buy things for which they cannot pay cash.

   d. Leadership, followership, respect for rights and opinions of others were developed.

   Nine 50-minute periods were used.

UNIT IV

VISIT A DISPLAY OF MONEY AT THE BANK

A Pupil's Account

The four eighth-grade mathematics classes visited the display of money in the lobby of the First National Bank as one of the activities for Pennsylvania Week.

All were taken into the vault. The men were kind in answering the many questions. The weight of the door was found to be twenty tons. We were shown just how it closes, how it is set to open at the proper time. Several of the boys were allowed to move the door. We were also shown the emergency door to the vault which has had to be used only twice since the bank has operated at its present location. The bank has been located at this place since 1914, but has been operating since 1856.

The safe deposit boxes were explained. There are three hundred which rent for from $3 to $25 a year, depending on the size.

We were also shown how fresh air is pumped into the vault at various times. This was especially necessary with more than twenty people in it at one time.

Deposit slips, checks, and other items were explained. We were privileged to see the people at work. Some of them took time to answer various questions, to stamp or cut "cancelled" on material on banking which was given to all pupils.

Shortly after the first contingent of pupils arrived, a reporter was there to take pictures. All tried to get a front seat so they could be seen in the picture.

Some of us saw pictures taken in 1914 showing the vault being brought to the bank by a mule team. What fun to see the old style automobiles on the street and the old-fashioned dresses of the men and women!

We learned something about the interest rate for personal loans and mortgages; and why the bank pays only 1 per cent on savings accounts.

The experience was very interesting and of much value. We were especially happy to have our English teacher with us. This can be an English as well as a mathematics project.

THE COMMON LEARNINGS PROGRAM

As implied in the title, the aim of this program is to provide the student with the mathematical competence which every citizen needs for successful everyday living. Although the committee was careful to build the program upon a definite set of mathematical topics, it also realized that a functional study of mathematics cannot be far removed from life activities. To this end, much attention was given to the indication of social situations or problems in which a mathematical concept or topic can be used.

Studies in the past have indicated that the average person has little use for the formalized program of algebra, geometry, trigonometry; so some individuals have concluded that there need be no mathematics beyond arithmetic. However, there is a mathematics program that will function in daily living. The purpose of this section is to suggest what that program should be. To plan a program well balanced between achieving mathematical power and the ability to sense the use of that power in social situations is a primary goal for every teacher of the Common Learnings Program. Another important aspect of this program is the attention given to grade placement of topics which facilitates, in so far as is practicable, the possible shifting of a pupil from one mathematical program to the other if a careful study of the pupil's educational program and aims and ability warrants such a change.

A Four-Year Program

The courses recommended to achieve the objectives of the Common Learnings Program are labeled Basic Mathematics I, II, III, and IV and are recommended for grades nine, ten, eleven, and twelve, respectively. It is possible that some schools are not ready to engage in such a comprehensive program of general education in mathematics at this time. Rather than attempt too much too soon, thereby inviting failure from the start, schools should study their own resources—teaching staff, teaching materials (textbooks, course outlines, experience units, etc.), and teaching experiences—before they inaugurate a four-year program in Basic Mathematics. Much good can come from a three-year program, although the committee urges the schools to work toward a four-year program and put it into operation as soon as conditions warrant. The committee favors a continuous program throughout each year even if it needs to be abbreviated to three periods a week, synchronizing it with music, health, band, or some other part-week subject program.

Teacher Selection

In preparation for offering Basic Mathematics, it cannot be too strongly urged that care be exercised in choosing teachers who are sympathetic with pupil purposes and needs and who have confidence that through these courses they can make a real contribution to the education of pupils. Successful teaching of the Common Learnings Program demands superior teachers. The teacher desirable for such courses has been described by Hawkins[1] as follows:

1. He is sensitive to the topics in mathematics that have value for all pupils.
2. He sees the interplay of arithmetic, algebra, and geometry as tools in solving problems and the symbols as aids in thinking.
3. He is less concerned with teaching certain algebra and geometry topics as such than he is with seeing pupils develop power in handling quantitative situations.
4. He is enthusiastic and likes young people. He is skillful in presenting subject matter and displays a sense of humor. He seems to have an infinite reservoir of patience, willing to reteach again and again if necessary.
5. He finds practical examples in the pupils' environment and in current publications to add reality to the class work.

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[1] George E. Hawkins, "Mathematics in General Education." *High School High Lights and Trends.* Scott, Foresman and Company, Vol. 7, No. 4 (May-June 1950), pp. 1-2.

6. He sees that pupils are required to recall and apply at intervals those topics which require thorough mastery. He arranges a testing program that enables both him and the pupils to observe progress toward their goal.

The curriculum for mathematical competence rests in a large degree upon the well-known *Check List* prepared by a committee of The National Council of Teachers of Mathematics. It is so basic to the work in this area that it is herewith reproduced in full.

CHECK LIST FOR MATHEMATICAL COMPETENCE

1. *Computation.* Can you add, subtract, multiply, and divide effectively with whole numbers, common fractions, and decimals?
2. *Per cents.* Can you use per cents understandingly and accurately?
3. *Ratio.* Do you have a clear understanding of ratio?
4. *Estimating.* Before you perform a computation, do you estimate the results for the purpose of checking your answer?
5. *Rounding numbers.* Do you know the meaning of significant figures? Can you round numbers properly?
6. *Tables.* Can you find correct values in tables; *e.g.*, interest and income tax?
7. *Graphs.* Can you read ordinary graphs: bar, line, and circle graphs? The graph of a formula?
8. *Statistics.* Do you know the main guides that one should follow in collecting and interpreting data; can you use averages (mean, median, mode); can you draw and interpret a graph?
9. *The nature of a measurement.* Do you know the meaning of a measurement, of a standard unit, or the largest permissible error, of tolerance, and of the statement that "a measurement is an approximation."
10. *Use of measuring devices.* Can you use certain measuring devices, such as an ordinary ruler, other rulers (graduated to thirty-seconds, to tenths of an inch, and to millimeters), protractor, graph paper, tape, micrometer caliper and thermometer?
11. *Square root.* Can you find the square root of a number by table or by division?
12. *Angles.* Can you estimate, read, and construct an angle?

13. *Geometric concepts.* Do you have an understanding of point, line, angle, parallel lines, perpendicular lines, triangle (right, scalene, isosceles and equilateral), parallelogram (including square and rectangle), trapezoid, circle, regular polygon, prism, cylinder, cone, and sphere?
14. *The 3-4-5-relation.* Can you use the Pythagorean–relationship in a right triangle?
15. *Constructions.* Can you with ruler and compasses construct a circle, a square, and a rectangle, transfer a line segment and an angle, bisect a line segment and an angle, copy a triangle, divide a line segment into more than two equal parts, draw a tangent to a circle, and draw a geometric figure to scale?
16. *Drawings.* Can you read and interpret reasonably well: maps, floor plans, mechanical drawings, and blueprints? Can you find the distance between two points on a map?
17. *Vectors.* Do you understand the meaning of vector, and can you find the resultant of two forces?
18. *Metric system.* Do you know how to use the most important metric units (meter, centimeter, millimeter, kilometer, gram, kilogram)?
19. *Conversion.* In measuring length, area, volume, weight, time, temperature, angle, and speed, can you shift from one commonly used standard unit to another widely used standard unit, *e.g.*, do you know the relation between yard and foot, inch and centimeter, etc.?
20. *Algebraic symbolism.* Can you use letters to represent numbers; *i.e.*, do you understand the symbolism of algebra–do you know the meaning of exponent and coefficient?
21. *Formulas.* Do you know the meaning of a formula–can you, for example, write an arithmetic rule as a formula, and can you substitute given value in order to find the value for a required unknown?
22. *Signed numbers.* Do you understand signed numbers and can you use them?
23. *Using the axioms.* Do you understand what you are doing when you use the axioms to change the form of a formula or when you find the value of an unknown in a simple equation?
24. *Practical formulas.* Do you know from memory certain widely used formulas related to areas, volumes, and interest, and to distance, rate, and time?

25. *Similar triangles and proportion.* Do you understand the meaning of similar triangles and do you know how to use the fact that in similar triangles the ratios of corresponding sides are equal?

26. *Trigonometry.* Do you know the meaning of tangent, sine, cosine? Can you develop their meaning by means of scale drawings?

27. *First steps in business arithmetic.* Are you mathematically conditioned for satisfactory adjustment to a first job in business; *e.g.*, have you a start in understanding the keeping of a simple account, making change, and the arithmetic that illustrates the most common problems of communications and everyday affairs?

28. *Stretching the dollar.* Do you have a basis for dealing intelligently with the main problems of the consumer; *e.g.*, the cost of borrowing money, insurance to secure adequate protection against the numerous hazards of life, the wise management of money, and buying with a given income so as to get good values as regards both quantity and quality?

29. *Proceeding from hypothesis to conclusion.* Can you analyze a statement in a newspaper and determine what is assumed, and whether the suggested conclusions really follow from the given facts or assumptions?

To achieve the desired mathematical competency with these abilities and to develop the power to apply them in life situations, much of the instruction should be handled through units related to or arising from life in the immediate community. Unusual or striking mathematical situations frequently reported in newspapers, news magazines, advertising posters and the like can start highly motivated investigations and reports rich in learning possibilities.

In addition, the spatial, quantitative, and time aspects of every task performed at work, at home, or during leisure time activity offer opportunities for application of mathematics. In every one of these aspects many units of measure need to be carefully defined and clearly understood. The thought that these units of measure must be used precisely if the expected results are to correspond with the predicted results has almost untold possibilities for useful learning experiences. This emphasis on precise measurements should induce pupils to rely very little upon subjective notions or guesses. Instead they will aim for the accuracy of mathematical expressions and scientific measurements. Frequently the difference between outstanding achievement and mediocrity, or even poor performance, in one's daily work is due to poorly developed understandings of the "real life" meaning of mathematical definitions, expressions, and measurements. The practices of cookery and photography are only two of many activities in which people engage where successful performance depends very largely upon the correct use of quantitative directions.

Lack of success in the performance of daily tasks by adults and pupils should be analyzed by the class under the guidance of the teacher with a view to finding the reason for faulty outcomes. A good slogan that a class may use to uncover or suggest activities for the basic mathematics class to probe into, is "Good directions followed precisely will always produce satisfactory outcomes."

Just as the understandings of and correct uses of spatial, quantitative, and time concepts are basic to production of good food and photography, as well as other work outcomes, so they are basic to:

a. The development and maintenance of good health and physical fitness
b. Understanding the economic problems of the community and family
c. Wise purchasing and use of goods and services
d. Appreciations of the methods of science
e. Wise use of leisure time
f. Careful thinking, writing, reading, and listening

Reference to the "Ten Imperative Needs of Youth" in Chapter I will suggest additional areas of common learning which will furnish topics and problems for Basic Mathematics.

Teachers and pupils should be continuously on the lookout for items that lend themselves to the analysis and study of life problems along these lines. Class reports, individual reports, production of notebooks and folders containing items pertinent to the mathematical study of individual or community problems should be supplemental to the more formal course outline.

To give point and more specific direction to the Common Learnings Program, it is recommended that *the needs of the individual* be emphasized in grade nine, *the interests of the earner* in grade ten, *the issues of the citizen* in grade eleven, and *the problems of the consumer* in grade twelve. Naturally, some problem material will be treated successively on several grade levels. Such applications to varying life situations and varying depths of penetration will help to make the learning more real, more thorough, and more comprehensive.

In schools where there are large numbers of dropouts by the end of the tenth year or where, for the present, a two-year sequence of Basic Mathematics seems feasible, consideration should be given to devoting a half year to each of these four areas for emphasis. Thus the high lights of the four areas will be presented for those pupils who will end their formal schooling by the tenth grade.

Since the concern for the vocational needs of pupils occupies a prominent place in the Common Learnings Program, in the secondary school teachers should not hesitate to divide the classes into relatively small groups to study materials common to related vocations.

Proper Guidance

The problem of who shall follow the Common Learnings Program and who shall follow the Special Mathematics Program is a very important one. No school, no teacher, and no pupil can afford to deal lightly with this matter. It deserves most careful study over a considerable period of time. Many a person has experienced keen disappointment when he has learned, after completing formal schooling, that he needed certain basic mathematics which he never studied simply because after grade nine he was permitted to choose, without any serious investigation, a program which did not include any training in mathematics. Let it be constantly borne in mind, that few, if any, studies are so difficult to "make up" or "get by oneself" as mathematics. The decision as to which program in mathematics a pupil shall follow is, therefore, an exceedingly crucial issue.

The decision should be made by considering a number of factors on which the student concerned has already furnished indicative evidence. These factors are:

1. The pupil's past record in arithmetic or in other mathematics courses
2. Teachers' estimates of a pupil's ability to profit by extended education. The phrase "extended education" is here used in a general sense and not necessarily as "mathematical education"
3. The pupil's mathematical and general aptitude as evidenced by objective aptitude tests
4. Parental interests, desires, and powers of support as related to the child's extended education

A rule-of-thumb procedure might be as follows:

> In order to help classify the pupils within a school, administer to all entering pupils an intelligence test and a general mathematics test for both of which norms based upon the same population are available. If a pupil's comparative norm on the mathematics test is appreciably higher than the comparable norm on the intelligence test, he might be classified as mathematically-minded; and, if other factors warrant it, he should be guided into the Special Mathematics Program.

If his normative score on the mathematics test is appreciably lower than his comparable score on the intelligence test, the pupil probably should take the Common Learnings Curriculum.

When a pupil's score on both of the tests falls approximately on the same level, other factors will have to provide the determining evidence.

The choice shall definitely not be made on the basis of whether or not the student is a prospective college student. Some prospective college students should follow one program; others should follow the other program. The choice should be made on consideration of what will provide the pupil with the most substantial stimulus for developing his own personality with respect to needed knowledge, facts, habits, attitudes, and experiences which are mathematical in nature. The making of a proper choice in this important matter calls for closest educational cooperation among the guidance, teaching, extracurricular and administrative forces of the school.

In the main, those pupils whose abilities, inclinations, and vocational leanings are toward areas in which mathematics and the physical sciences are basic will follow the specialized program. We would find in this program those pupils interested in teaching mathematics, in scientific careers, in research, in engineering, or in some other line of technical work. Those students who are unusually talented and interested in mathematics will want to and should enroll in this program for their own satisfaction. All the other pupils should enroll in the Common Learnings Program which will emphasize the mathematics of everyday living.

Since it is practically impossible for pupils and their counselors to foretell or foresee with precision the vocational needs of pupils as they progress through school, there is likelihood that some pupils who start in one of the mathematics programs will want to shift to the other. Since the Specialized Program is highly sequential and the Common Learnings Program is directed at the more practical aspects of mathematical problem solving, it is much less difficult for a pupil to shift from the Specialized to the Common Learnings Program than it is for the pupil to shift in the opposite direction. The pupil who has elected the Common Learnings Program may have such a change of plans or interest that the Special Program is more suitable for his purposes. Although he may be handicapped in trying to move from socialized to formalized mathematics, an individual possessing mathematical aptitude might transfer by one or a combination of the following plans:

1. Self-study—possibly correspondence work
2. Doing extended work under the direction of a tutor or a Common Learnings teacher
3. Taking another year of secondary school training

Transfers to the Special Mathematics Program can probably be made most effectively at the end of grade 9 or grade 10.

Suggested Courses in Basic Mathematics

It is here proposed that the four areas described in Basic Mathematics be considered a Common Learnings Program for all pupils not enrolled in the traditional sequential courses which are referred to as the Specialized Mathematics Program. In addition, these areas is Basic Mathematics should be looked upon as resource areas for making the specialized mathematics courses more meaningful and for giving to those pupils enrolled in the latter courses some understanding of mathematics useful in the daily living of every citizen and consumer.

One of the reasons why this manual is called a tentative guide is that some of the experimental courses, like those in Basic Mathematics, need to be studied and tried out before the courses can be outlined in a more definite form. The suggestions herein presented represent some preliminary ideas which may serve to demonstrate the purposes and the nature of a Common Learnings program in mathematics.

Many teachers believe that there is a scarcity of textbooks for courses in Basic Mathematics. Although there is a considerable amount of truth in this belief, a combination of textbooks, workbooks, mimeographed lesson plans or study guides, notebooks, and community resources can be of great help. Furthermore, suitable textbooks are rapidly becoming more numerous. Even the conventional mathematics textbooks can be used satisfactorily if the more functional topics are studied.

Since the Basic Mathematics courses will attract students from the entire range of student ability, special care needs to be exercised so that the abler students will be challenged and have their needs fully met. The abler students may assist as student leaders or teachers; they may conduct research studies and make reports to the class or teacher, or both; they may pursue separate assignments, prepare notebooks, models, exhibits, etc.

Although the Check List for Mathematical Competence, page 101, probably lists all of the important mathematical topics needed in ordinary life activities, the courses in Basic Mathematics should present the topics in so far as is possible in connection with life problems. Broad topics like "Mathematics in Home Building," "The Mathematics of a

School Carnival," "The Mathematics Needed to Operate a Garage," "The Mathematics in Baking a Cake," and the like may serve as beginning units. After the larger topics are studied and the pupils become alert to the detection of quantitative concepts needed to study a problem, smaller interesting subtopics will become apparent. Pupil interest will depend in a large measure upon the teacher's enthusiasm and upon the extent to which the activities are related to daily living.

With this philosophy in mind the following introductory description of the essence of each of the four phases of Basic Mathematics for the secondary school is presented as a suggested outline. The areas mentioned in each year are for the guidance of the teacher and need not be developed in the way or in the sequence presented.

BASIC MATHEMATICS I

GRADE NINE

For the Individual[1]

Area of Activities

This phase of the program lays stress upon the role of mathematics in personal security and development, with indirect attention to the earner, the citizen, and the consumer. Guidance is the key to this year's work. The pupil should take stock of his own mathematical competence by using the Check List found on page 101.

Basic mathematics for the individual involves:

1. REVIEW AND EXTENSION OF COMPUTATIONAL SKILLS

    This is part of the program in every area of the mathematics program, but, to be effective, it must grow out of use and not be taught as isolated drill. For example, an experiment with a ninth grade class in "Aviation in General Mathematics" revealed that pupils readily learned "fundamentals" when presented in a setting of mathematical significance.[2]

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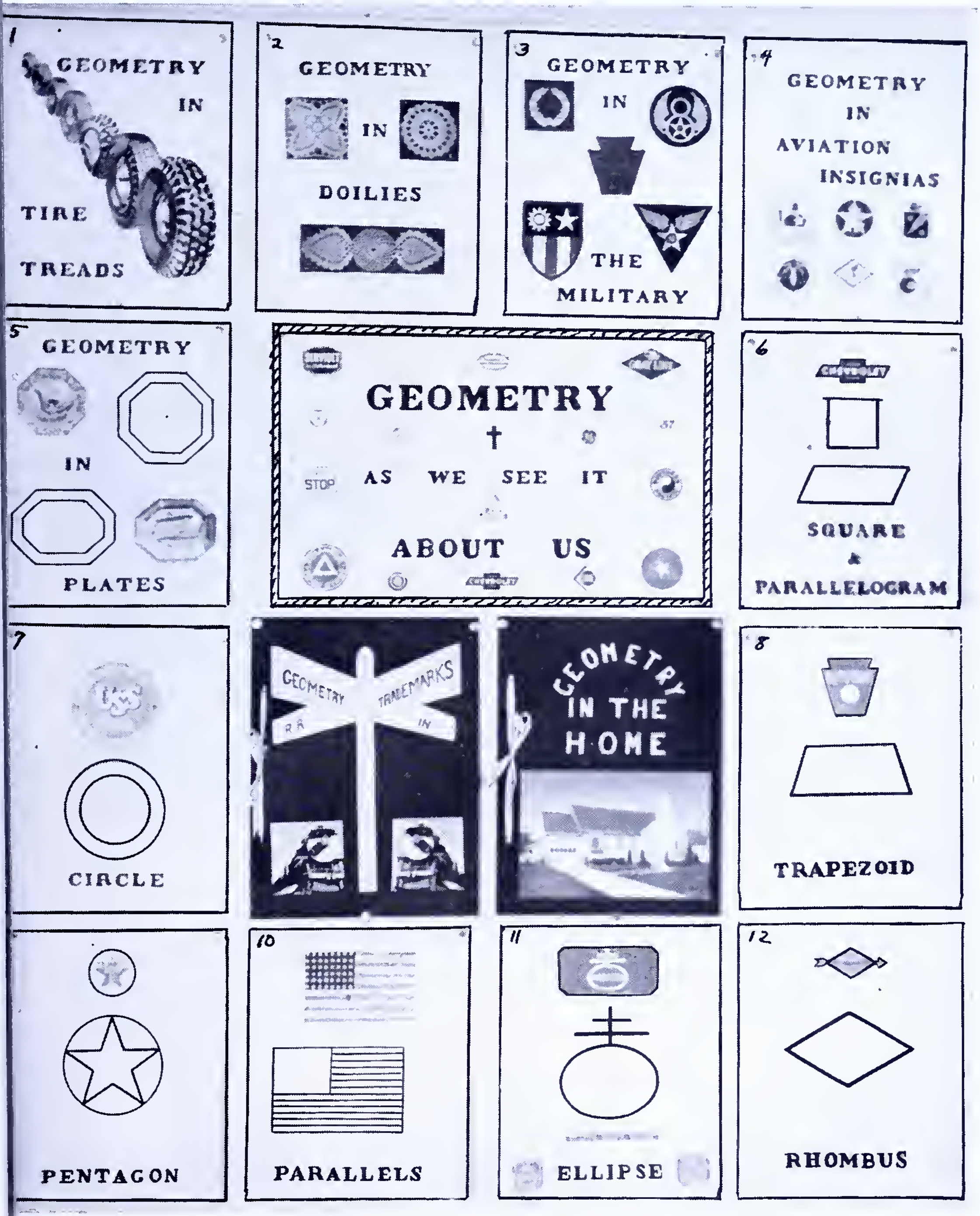
[1] This section provides an excellent opportunity for "articulation" since some of the topics may well be developed or even introduced in other classes. The important consideration is that the teacher pay attention to these needs of the pupil in his ninth grade program.

[2] See *Bulletin of the National Association of Secondary-School Principals*, Vol. 30, Jan. 1950, pp. 108-119.



MATHEMATICS IN COMMERCIAL ART

Functional Competencies

TOPICS	SUGGESTED ACTIVITIES
1. Addition, Subtraction, Multiplication, and Division of Integers	a. Use inventory tests (pretests).
	b. A student may give a report on the history of numbers.[1]
	c. From day to day, place a mathematical puzzle on the board. This should involve the use of fundamental operations.
2. Addition, Subtraction, Multiplication, and Division of Fractions	a. Write the answers to homework problems on the board so that pupils may check them as soon as they arrive. The teacher may then walk around the room to spot difficulties.
	b. Have individual reports on the historical development of fractions for the interest of all.
	c. Show the advanced group in the class various short cuts in handling fractions.
3. Addition, Subtraction, Multiplication, and Division of Decimal Fractions	a. Show filmstrip on fractions, decimals, and percentage.[2] The filmstrip will also review fractions and at the same time serve to introduce the next unit.
	b. Pupil reports on history and development of the decimal point. "Zero" would be a pertinent topic at this point also.
	c. Give each pupil a personal decimal card.[3]
4. Percentage and Problem Solving	a. Use supplemental practical problems to enliven the material in the text.[4]
	b. Have various pupils check with local businessmen on the need for understanding percentage and report to the class.

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[1] David E. Smith, and Jekuthiel Ginsburg. *Numbers and Numerals.* New York, Teachers College, Columbia University. 35 cents postpaid. A good reference for students.

[2] *Fractions, Decimals,* and *Percentage.* 59 frames. Jam Handy. Develops whole numbers to fractions, decimals, percentage and back. Tells when it pays to use a decimal point instead of a fraction, and when fractions are most practical. Emphasizes the importance of selecting the right form.

[3] L. W. Starrett Co., 17 N. Jefferson St., Chicago, Ill. *Decimal Equipment Chart.* Large. Free to teachers on request. *Decimal Cards.* Small. Free.

[4] *Railroad 'Rithmetic, Book II, Supplementary Railroad Transportation Problems for Secondary Schools, 1950.* Baltimore and Ohio Railroad Co., Baltimore 1, Md. Units 1, 2, 3, and 4. pp. 1-6.

TOPICS	SUGGESTED ACTIVITIES
	c. Have class members bring in newspaper articles showing how they use per cent.
	d. Show a filmstrip on the use of percentage.[1]
5. Denominate Numbers, Ratio and Proportion	a. Show filmstrip on ratio and proportion.[2]
	b. Use and solicit mathematical puzzles involving ratio and proportion.
	c. Take the class outside to compute the height of various objects by the shadow method.
	d. For the reading table: *Story of Weights and Measures.*[3]

II. ROUNDING OFF NUMBERS[4]

A. All measurements are approximate and therefore "rounded." Illustrations: The classroom temperature is 72 degrees—The recently fallen snow measures five inches—My age is 14 years. On the other hand, numbers determined by counting are not approximate but exact. Example of exact numbers: Our school building has 12 classrooms, 450 locks, 12 classroom clocks, and employs 4 day-time janitors.

B. Estimates are rounded numbers:—The newspaper estimates an an attendance of 800 people at the ball game.

C. Large numbers are frequently rounded to the nearest thousand, million or billion.

PUPIL ACTIVITIES:

1. Have four pupils measure the width of the classroom as accurately as possible, using a ruler, a yardstick, a tape measure, a carpenter's rule, and report their measurements separately.
2. Make a list of exact numbers and a list of approximate numbers found in one issue of *Life* magazine, *Better Homes and Gardens,* or the like.

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[1] *Using Percentages.* 23 frames. Curriculum Films. Gives practice in changing a per cent to a decimal and to a fraction, finding a per cent of a number, finding what per cent one number is of another, and finding a per cent of changes.

[2] *Ratio and Proportion.* 50 frames. Jam Handy. Develops graphic visualizations of what is meant by ratio and proportion and shows their usefulness in practical everyday situations.

[3] *Story of Weights and Measures.* Committee on materials of instruction, American Council on Education, 5835 Kimbark, Chicago.

[4] See Chapter VII in Boyer, *An Introduction to Mathematics for Teachers.*

3. Find ten measurements in journals that are expressed to the nearest unit, the nearest hundred, the nearest thousand, or the nearest million.

III. VOCATIONAL ORIENTATION

Study of vocational fields where mathematical training is of value, if not a prerequisite. (Note that three-fourths of all workers earn their living in 150 major occupations.)

A. *Guidance Pamphlet in Mathematics for High School Students* —Final report of the Commission on Post-War Plans of the National Council of Teachers of Mathematics (Published by *The Mathematics Teacher*)

B. Using Mathematics in Farming (for appropriate areas) Measurement and quantitative problems such as: Ordering lumber, measuring grain, marketing produce; building sheds, silos, fences; mortgage payments

C. A survey of the mathematical needs of workers in local businesses and industries.

IV. DEVELOPMENT AND USE OF A SATISFACTORY PERSONAL BUDGET PLAN

A. *Attention to objectives* such as:
  1. Understanding the value and importance of budgeting
  2. Understanding the values, limitations, and uses of model budgets with general percentage figures for expenditures
  3. Learning how to set up a budget suitable for a particular individual's needs

B. *Study of personal problems* such as:
  1. Should I have an allowance? How much?
  2. Should I work to supplement the family income?

C. *Wise use of leisure time.* (See illustrative resource unit on mathematics and leisure, Chapter II, pages 55-58, section on units.)
  1. For a period of one week keep an accurate account of how your time is spent each day. Carry a small card or notebook with you to keep this daily account. At the end of the week add up the total and calculate the per cent of time spent in various types of activities.

2. Automobiles represent one of the greatest expenditures of the American people for recreation. If your family owns one, compute the total yearly cost of this recreational facility. In arriving at this cost make sure to include the following items:

    (a) Initial cost
    (b) Depreciation
    (c) Garage
    (d) Fuel
    (e) Repairs
    (f) Insurance
    (g) Taxes
    (h) License fees

3. Make a map showing parks and other facilities for recreation in your locality and state. List the cost involved in using them.

D. *A month of practical arithmetic.* (See Grade 8, Unit I, page 96).

E. *Visual aids*

1. *Your Thrift Habits.* 1 Reel. B. and W. or Color. Coronet.
2. *Consumer Protection.* 1 Reel, B. and W. or Color. Coronet.

F. *Bibliography*

1. *Managing Your Money.* Consumer Education Study. National Association of Secondary-School Principals. 1201 Sixteenth St., NW, Washington, D. C.
2. Stewart, Maxwell S., *Women and Their Money.* Public Affairs Committee, Inc., 22 East 38th St., New York 16, N. Y. $.20.

V. INTRODUCTION TO PRACTICES IN COMMON USE IN BUSINESS TRANSACTIONS

A. Business forms, making of invoices, simple accounts, credit, discounts, financing business operations

B. Use of rapid calculation—short methods of multiplication and division

C. Reading and making of graphs of cost, overhead, profit

D. The travels of a check

E. Talks by local businessmen on training needed by their employes

F. Using business services
Personal shopping

VI. MATHEMATICS AS A FORM OF COMMUNICATION

A. The "shorthand" of algebra

B. See topics in Algebra I marked "Basic," pages 194, 195

VII. THE FORMULA[1]

A. Discuss the prevalence of the use of symbols in modern society. Such symbols as S.O.S., etc., m.p.h., A.M., U.S.A., and others suggested by the students may be mentioned. The idea that symbols are useful as convenient shorthand devices should be developed. Familiar symbols used in mathematics, such as +, −, ×, ÷, etc., may also be mentioned.

B. Now propose as a problem in a real setting, the finding of the perimeter of a triangle. The procedure developed in solving this problem may be stated as a rule. By the use of appropriate symbols suggested by the pupils this rule may be stated as a formula.

Other situations may be treated similarly, *e.g.*, Selling Price = Cost + Gain; Distance = Rate × Time; Average $= \frac{a + b + c}{3}$, etc. In each case other problems than those used to derive the formulas should be solved by the use of the formulas. Thus the concept of the formula as a generalization and its application in many situations of a similar nature can be developed.

C. Another method of deriving formulas involves the use of tables. Tables such as the following may be used

N	1	2	3	4	5	6
C	\$.26	\$.52	\$.78	\$1.04	\$1.30	\$1.56

In this table N represents the number of gallons sold by a gas station attendant and C represents the cost of the gasoline sold. The table should be prepared by the members of the class, the relationship stated in words, and the verbal statement reduced to a formula. Many such situations will suggest themselves to the teacher.

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[1] Adapted from *Teacher Notes on Ninth Year Second Track Mathematics*—prepared by a committee of New York City teachers.

D. A third method of arriving at a formula involves simple experimentation. For example, the number of triangles into which a polygon may be divided by drawing diagonals from one vertex is a fruitful experiment. Another such experiment involves the derivation of the circumference of a circle. This can be done by collecting data with respect to the ratio of the circumference of a circle to its diameter. The measurement of a number of objects such as the top of a waste paper basket, a tumbler, a jar, a tin can, etc., will yield clarifying results. A third experiment involves the derivation of formulas for the areas of a rectangle and an isosceles right triangle. In both these cases the area can be found by counting squares on graph paper.

E. After these formula experiences, more difficult situations may be proposed by the teacher.

1. The Christmas Club formula $S = \$5 + .75W$ where S represents the sum and W the number of weeks of payment is an attractive situation for pupil analysis.
2. The formula $P = F + 2G$ for computing points in a basketball game is simple and interesting in this connection.
3. Another formula of this type is $R = .65A + .35C$ where R represents total receipts, A equals the number of adults and C the number of children attending a moving picture theater.
4. If the class seems interested it may profitably investigate the formulas involving taxicab charges and circulating library fees.

F. The next step in the formula unit involves formulas which are useful but too difficult to derive and must therefore be supplied by the teacher. The formula for the volume of a cylinder, the conversion formula from centigrade to Fahrenheit, and the interest formula are examples of these.

G. In all this work on the formula, meaningful problems, including those involving fractions and decimals, should be proposed and solved. This work will necessarily involve only simple substitution.

Although perimeters, areas, and volumes are mentioned above incidentally, it is expected that these concepts will be developed carefully and that numerous problems involving these concepts will be proposed and solved. Composite figures involving combinations of the rectangle, triangle, and circle are interesting to students and provide excellent review of fundamentals. Similarly problems in three dimensions involving the rectangular solid and the cylinder provide opportunities if pupils are interested.

H. If the formula unit has aroused sufficient interest, additional formulas to be used only for inspection and general discussion may be proposed by the teacher or by the students. Formulas from science, such as the law of the lever, formulas used in industry taken from machinists', engineering, or aviation handbooks may be used in this connection.

VIII. THE MATHEMATICS OF GENERAL INFORMATION

A. Develop familiarity with the meaning and use of basic mathematical terms in everyday use. These would include: unit, ratio, average, percentage, proportion, rate, variable, constant, dependence, positive, negative, formula, dimension.

B. Be able to read and interpret very large (billions of dollars) or very small (millionths of an inch) numbers as they appear in written form.

IX. THE MATHEMATICS OF PERSONAL HEALTH

A. Buying health services: Study group insurance plans—availability, coverage costs. Investments of tax money to improve the health of the nation.

B. The costs of illness—a problem for pupils:
List the number of times that you and the other members of your family have been sick in the past year. What were the causes? How many days of school or work were lost? Estimate the total financial loss to your family. (Include both the medical costs and loss of work.)

C. Mathematics of diet—calories, weight and height charts, measurements in cooking.

X. MAPS AND CHARTS

A. Understanding types of maps and charts and their uses

B. Reading scales and legends on maps and charts

C. Finding position on maps and charts

1. Coordinates—use of intersecting lines of position to locate points on the earth's surface: crossroads and city streets systems, grid system for military maps, longitude and latitude.

2. Meridians and Parallels—measurement of longitude from the Greenwich Meridian, and latitude from the Equator in degrees, minutes, and seconds of arc. Locate major cities in terms of their latitude and longitude.
3. Time Zones—comparison of arc time in terms of degrees, minutes, and seconds with clock time in terms of hours, minutes, and seconds (15° of longitude equals one hour); establishment of time zones and the International Date Line (180° meridian). Discuss need for resetting watches on long trips east or west.

D. Measuring distance on maps and charts
   1. Statute Miles—use of scale to determine statute miles (5,280 feet) on maps and charts; discussion of use and advantages.
   2. Nautical Miles—comparison of nautical mile (6,080.2 feet, or one minute of arc on a meridian or the Equator or any great circle) with the statute mile (5,280 feet); discussion of use and advantages.

XI. PROTECTING VALUABLES
   A. Insurance in mailing
   B. Insurance of personal goods
   C. Safety deposit boxes

BASIC MATHEMATICS II

GRADE TEN

Mathematics for the Earner

Areas of Activities

This phase of the program lays stress upon mathematics for the earner, with subordinate attention to the individual, the citizen, the consumer. Suitable topics should include: PAYMENT FOR SERVICES ON THE JOB, KEEPING ACCOUNTS, and MEASUREMENT ON THE JOB.

I. PAYMENT FOR SERVICES ON THE JOB[1]
   A. *Bases for payment for services*
      1. Time basis: hourly, daily, weekly, overtime
         a. Make a study of the local community. Determine the various bases for payment and secure data on prevailing rates of pay. Using the various rates, compute the wages

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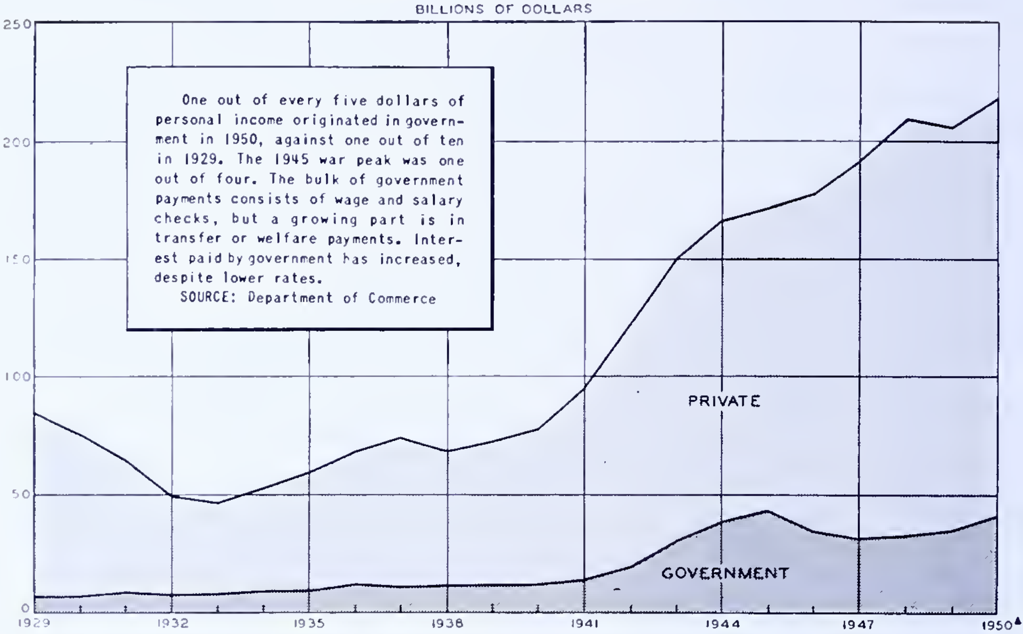
[1] Adapted from *Tentative Guide to Mathematics for the Worker,* Philadelphia Public Schools.

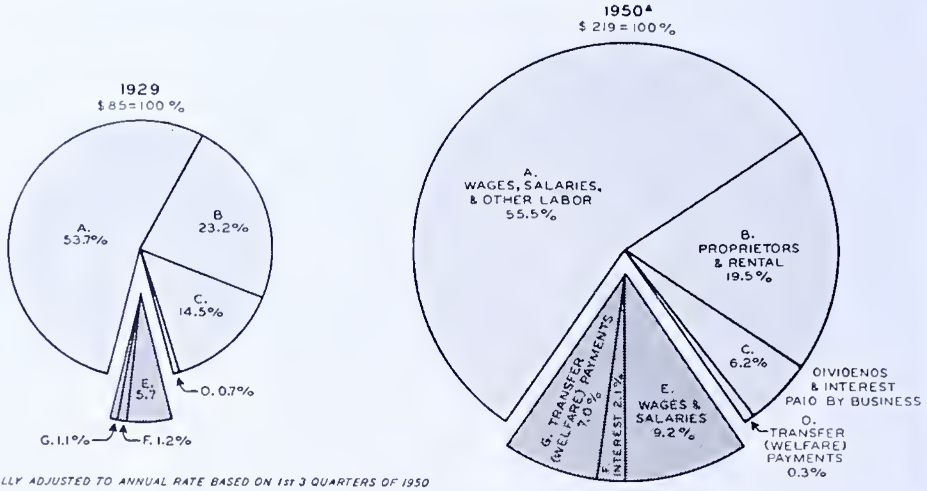
SOURCES OF PERSONAL INCOME
UNITED STATES, 1929-1950



One out of every five dollars of personal income originated in government in 1950, against one out of ten in 1929. The 1945 war peak was one out of four. The bulk of government payments consists of wage and salary checks, but a growing part is in transfer or welfare payments. Interest paid by government has increased, despite lower rates.

SOURCE: Department of Commerce

COMPOSITION OF PERSONAL INCOME



▲ *SEASONALLY ADJUSTED TO ANNUAL RATE BASED ON 1ST 3 QUARTERS OF 1950*

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for certain time periods, such as a 40-hour week, a 52-week year. What jobs are seasonal and what financial security can a worker get from them?

b. Workers paid on time basis:
Example—Automobile Service Station Attendant

Automobile Service Station Attendant[1]

Hourly rates (study current practices in the local community)

For experienced workers—range between 75¢ and $1.25 per hour
For beginners—from 50¢ to $1.00 per hour

Weekly Earnings

In many large cities, the average attendant earns from $45 to $50; in smaller cities, weekly wages are approximately $5 to $10 lower. Higher earnings generally come as a result of longer working hours. In addition to wages, attendants may receive commissions on sales. Bonus and incentive pay systems are used in some stations. Earnings of operators and owners depend upon location, size of station and volume of business, and therefore vary greatly.

Suggested Problems

John Clark works at a service station from 8 a.m. to 4 p.m. and receives $25 per week for a 5-day week. If he works overtime, he receives $1.00 per hour for each hour worked. If he works on one of his days off, he receives double time. During one week he worked the following schedule of hours:

Sunday 8 a.m. to 5 p.m.
Monday 8 a.m. to 5 p.m.
Tuesday day off
Wednesday 8 a.m. to 5 p.m.
Thursday 8 a.m. to 3:30 p.m.
Friday 8 a.m. to 6 p.m.
Saturday 8 a.m. to 6 p.m.

How many hours of overtime did he work?
How many hours of double time did he work?
What was his week's pay?

If John were offered another job at a different service station which paid 95¢ an hour, would he be better paid at this new job if he worked the same number of hours?

2. Piece work: Factories, work taken home, fruit and vegetable picking.

   Make a chart comparing the earnings of a worker on a piecework basis with his earnings on an hourly basis.

3. Commission: Straight commission, commission plus expenses, salary plus commission.

   Using current commission rates, compute the amount of earnings of local workers such as milkmen, newsdealers, insurance salesmen. Secure copies of a milkman's account sheet.

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[1] From *Occupational Guide* issued by the Social Security Administration, Bureau of Employment Security.

4. Annual salary.

   Given the annual salary of a number of selected occupations (check local or state employment offices), compute the monthly, semimonthly, and weekly payments.

5. Supplementary payments: Maintenance, tips, bonus, profit sharing, vacations with pay, medical services, employe discounts.

6. Individual enterprise: One's own business.

   As newsboys, baby-sitters, errand-runners, etc., pupils may already be in part-time businesses for themselves. Consider also the neighborhood "home owned" stores and businesses as well as professions such as medicine, dentistry, music.

   Study per cent through problems about selling price, cost, profit, discount.

   Become familiar with business forms such as inventory blanks, checks, deposit slips, bank statements, income tax blanks, bills. Make graphs to show distribution of cost, overhead, profit.

B. *Determination of "take home pay"*

1. Take typical earnings and show how the pay check "shrinks" by such deductions as
   - a. Government deductions: Federal income tax, city wage tax, social security payments
   - b. Voluntary deductions: union dues, group insurance (health, accident, hospitalization, medical care), pension plans, saving plans, Government bonds, company stock, Christmas funds
2. Find the length of time required to purchase bonds of specified denominations when the periodic payments are given. From U.S. Treasury offices secure pamphlets of School Savings Plans.

C. *Other factors to consider in job selection*

1. Occupational expenses: transportation, tools or equipment, special clothing or uniforms. Select specific occupations of pupil interest and make lists of these items and their cost.
2. Occupational benefits: in-service training, recreation, environment (outdoors), unemployment protection, pension plans, steadiness of employment, freedom from hazards.
3. Calculate the monetary value of these benefits to the employes.

D. *The history of present monetary system of payment*

See the American Council of Education, *Achievement of Civilization Series,* for some stories of historical background of the development and the uses of numbers.

II. KEEPING OF ACCOUNTS

A. *Study and use of forms* used by the employer to indicate employe earnings, deductions, net pay

B. *Simple accounts of small businesses*

1. Learn to define and to calculate
   a. Accounts receivable and accounts payable
   b. Purchases and sales
2. Realistic experience in the importance of accuracy in computing percentages and totals of different forms and in the careful tabulation of data can be gained through practice with common business forms, such as: cash book, sales journal, trial balance, ledger, bill of lading, credit memorandum, invoice, voucher, sales slip, money order
3. A form of valuable "work experience" can be gained by having groups or individuals set up forms and determine profit and loss over a set period for the cafeteria, school bookstore, or student organizations.

III. MEASUREMENT ON THE JOB[1]

A. *Nature of measurement*—accuracy and precision, possible error

The accuracy of a number depends on the number of significant figures it contains. Thus 4 feet, 16 inches, 1.6 feet, 0.4 inch, and 4,000,000 miles all have the "same accuracy." Of these numbers, 4 feet and 16 inches are "equally precise" for they are both expressed to the nearest unit. Likewise, 0.4 inch and 1.6 feet. Both are expressed to the nearest tenth of unit.

The "possible error" of the measurement 4 feet may be 0.5 foot if the expression 4 feet is used to refer to a distance *between* 3.5 feet and 4.5 feet. Thus the relative error in the measurement 4 feet would be: $\frac{0.5}{4} = \frac{1}{8} = 12\frac{1}{2}$ per cent.[2]

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[1] Adapted from *Tentative Guide to Mathematics for the Worker*, Philadelphia Public Schools.

[2] Teachers will find helpful information on this topic in L. E. Boyer, *An Introduction to Mathematics for Teachers,* pp. 83-100.

B. *Examination of measurements involved in familiar occupations*

Examine with the pupils several familiar occupations and analyze them with respect to: measurements performed, instruments used, and concepts involved.

It should be noted that in jobs such as carpentry and dressmaking, measurements of length are used directly as taken. In certain other jobs, for example, painting, plastering, and roofing, the determination of areas is foremost in the contractor's mind. Likewise volume is of paramount importance in the work of the excavator, the masonry contractor, and the heating and ventilating engineer. However, linear measurements (length, width, height) are basic and their accurate determination must precede any calculation of areas and volumes.

SUGGESTED OUTLINE FOR JOB ANALYSIS

Job	*Measurements Performed*	*Instruments Used*	*Mathematical Concepts*
Contractor Carpenter Painter Roofer Plasterer Mason	Measuring lengths, widths, heights Determining areas and volumes Estimating materials Figuring costs	Folding rule Architect's scale Tape measure Carpenter's square Protractor Dividers Calipers	Length Area Volume Tolerance Angular measurement Drawing to scale Equality
Nurse Medical technician	Taking temperature Measuring drugs Measuring height and weight Counting pulse	Thermometer Scales and balances Graduate Height meter	Degree Per cent Cubic centimeter (c.c.)

C. *General applications of measurement*

1. Measures of length, area, volume
2. Tables of measures
    a. English
    b. Metric
    c. Relationships between English and metric; use of problems making simple comparisons, such as 35-mm films, 105-mm guns
3. Measuring instruments
    a. Familiarity with instruments in common use—precision instruments for those interested in shop work
    b. Ability to use instruments when needed

4. Formulas of length, area, volume as aids in solving problems dealing with occupations
5. Meaning of tolerance
6. Measures of capacity and weight
7. Measure of temperatures
   Use of signed numbers in thermometer readings
8. Measures of time
   a. Clock
   b. Time belts
9. Use of meters
   a. Household utilities
   b. Parking meters
   c. Various types used in radio and electrical work

IV. PROBLEMS OF THE EARNER

A. *Mathematical skills and concepts*

In solving the problems of the earner, certain mathematical skills and concepts are necessary. For example, twenty-two business organizations, labor union educational advisors, and personnel managers were asked to express themselves as to the type of mathematical training they thought would best serve the young high school graduate as he enters the business and social life of the community. Walter A. Wittich, Public School Supervisor, Madison, Wisconsin, in his article, "School and Community Look at the Content of Consumer Mathematics," lists the following as some of the items given most often by those questioned:

1. Linear measure
2. Quick computations
3. Installment buying, carrying charges, cost of loans
4. Ability to make change up to $10
5. Ability to compute per cent mark-ups as ordered by store managers
6. Ability to compute amounts of materials such as paint, wood, sand, etc., to be used on the job
7. Ability to keep accurate record of hours worked, material used, etc.
8. Knowledge of liquid and dry measures
9. Ability to compute unit and quantity prices as given by grocery stores

10. Ability to write numbers correctly
11. Ability to interpret charts and tables
12. Ability to make inventories and add and subtract
13. Ability to compare values in canned goods by comparing weights and prices

These men felt that if their employes could handle the problems involving these abilities, they would be better than average workers.

B. *Competence of workers*

An analysis of the activities of workers will reveal that mathematical competence for an efficient worker needs to include the the following:

1. Basic operations with whole numbers, fractions, decimals

   In modern machine work a thorough working knowledge of fractions is imperative. An example is the use of fractions in tolerance which are the small permissible differences in dimensions. Charts of decimal equivalents of the fractional parts of an inch should be provided.
2. Percentage
3. Use of tables
4. Graphs—stressing of ability to interpret graphs[1]
   a. Using the attendance records of the class, prepare a graph of daily attendance (broken line graph). Have pupils discuss the answer to such questions as: (1) How has the attendance changed? (2) Explain the low points. (3) Explain the high points. (4) How many pupils were in attendance on ____________? (5) How would the graph change if the scales were changed?
   b. Such graphs should be concerned with current events, sports data, economic trends, such as prices, wages, national income.
   c. Have the class engage in the project of collecting statistics on the heights of pupils in class, making a distribution graph in the form of a bar graph. Have the class discuss the choice of a vertical number scale to represent heights. *Note:* Bar graphs must be constructed with

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[1] Procedures adapted from *Teacher Notes on Ninth Year Second Track Mathematics*, prepared by a committee of New York City teachers.

continuous scales starting with zero so that ratios may be pictured accurately. Discuss the *average height* as the arithmetic average; the *median height* as that height which represents the tallest of the first 50% of the class, when arranged from shortest to tallest.

d. Repeat step c with such items as surveys-of-opinion polls.

e. Introduce pictograms. Subject to a treatment similar to that of bar graphs.

f. Introduce broken line graphs. In this case the vertical number scale may be discontinuous so as to permit the graphing of large numbers in a small range. The purpose of the broken line graph is to note *changes,* while the bar graph and pictogram are used primarily to compare discrete quantities. Suitable data for this purpose are: (1) temperature readings, (2) the day-by-day average of a baseball team, (3) the fluctuations in a price index, (4) the changes in weight of an individual from year to year, and the like. Suitable questions are: Is it increasing or decreasing? What is the largest value? Will it rise or fall? At what time is it largest? smallest? What is the value at a time not given in the table (interpolation) ?

g. The reading and construction of circle graphs. These should be used only when an entire amount is divided into parts, such as a family budget, a city budget, the use of 24 hours by one pupil, etc.

h. Formula graphs. Suitable graphs are $p = 4s$ (perimeter of square) ; $c = 10n$ (cost of *n* articles at 10 cents each) ; $p = 2c$ (profit of 200%) . In constructing the graphs, first have class make a table of values for representative instances, then discuss the choice of units for the number scales, plot the points, draw the graph, and finally use it for quick readings as substitute for computations.

i. In teaching coordinate systems, do not overburden the topic with too many strange terms.

Introduce with the problem: I wish to meet a friend at a particular spot. How can we describe such a point so that there will be no doubt as to its position? In the ensuing discussion the use of *streets and avenues* as a coordinate system will develop.

In the same way *seats and rows* will locate the position of a pupil in a classroom, or a point on a map may be found at the intersection of a numbered line and a lettered line. Such a method of locating the position of a point is called "Rectangular Coordinates." These consist of two straight-line axes perpendicular to each other. This method applies to flat surfaces.

5. Ratio
6. Nature of measurement
7. Use of measuring devices
8. Use of tables of measure
9. Formulas—especially those used in business
10. Square root
11. 3-4-5 relationships in triangles
12. Problem-solving techniques[1]
    a. A few suggestions for the problem-solver

    Problems must be read slowly many times. Use a dictionary to acquaint yourself thoroughly with the meaning of every word. Check in the textbook the meaning of any technical term that is hazy. Discuss the meaning of the problem. Make a sketch (perhaps a scale drawing) of the pattern of the problem. List the given factors and the part to be found. Estimate the answer. Consider how the part to be found can be established from the given data; or how the given data can be made to yield the part to be found. (Sometimes it is easier to reason from an answer towards the known parts; at other times it is easier to reason from the known parts towards the answer.) Check your work. Prove your answer.

    b. A few queries for the unsuccessful problem-solver

    Can you state the problem in your own words without reference to the textbook, mentioning all given features and part or parts to be found? Did you start to outline on paper the conditions of the problem and the method of solution? Just where in the analysis of the problem or in the steps for solution do you fail to have confidence in your course of procedure? What makes you uncertain at this point? Do any of the former sections of the textbook give you an explanation on this point? Could there be several interpretations (and possibly several methods of solution) to the issue concerning which you are in doubt? If the teacher were not available, who could supply you with the necessary information? Have you thought about the problem sufficiently long? Did you prove your answer (and sometimes the textbook's answer too) right or wrong? If you were an adult, and not in school, how would you secure the correct answer to your problem?

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[1] Boyer, *An Introduction to Mathematics for Teachers*, pp. 127-128.

13. Estimating

V. VOCATIONAL CHOICE

Each pupil should select one vocation (reviewing those briefly surveyed in Basic Mathematics I) and study it intensively throughout the year with attention especially to the mathematical implications and problems.

VI. EXPANSION OF MATHEMATICAL HORIZONS

Refer to those topics marked (Basic) in Algebra for the Second Year, and develop the basic understandings of the topics thus marked. (See page 204.)

ILLUSTRATIVE UNIT

HOW SHALL WE USE A BANK?

1. *Introduction*

a. Procedure:

This unit should be discussed with the class and the objectives made clear. Adequate illustrative material should be used to develop each of the contributory objectives. A pretest is given to determine the pupils' knowledge of banking services and to help the teacher decide what to emphasize is assisting the pupils. During the early stages of work on the unit, the teacher should be able to discover the particular difficulties experienced by each pupil. Throughout the entire unit the teacher should be looking for incentives that will help motivate the work. Pupil understanding and attainment will be checked at intervals. At the conclusion of the unit, a final test is given to discover to what extent

the pupils have attained the central objective. Wherever necessary, special remedial work will be given.

b. Overview:

Some authorities trace the word *bank* to the "banc" or bench where the early money changers kept their coins. Others maintain that it is derived from "banck," the German name for a joint stock fund, which was converted by the Italians into "banco," meaning a heap or accumulation of money or stock. In colonial days in Massachusetts the issue of paper money was referred to as "raising a Banke," *bank* meaning the money rather than the institution which put it into circulation.

During the Revolutionary War there was urgent need for some means of financing the struggle. In order to carry on the war successfully, it was necessary that the revolutionary leaders have enough money to buy supplies and munitions to help support the less wealthy colonies. To provide for this emergency, the Bank of North America was established in 1781. The bank was granted a charter by the Commonwealth of Pennsylvania, and it continued to operate under a state charter until it became a part of the national system. This was the first bank organized in this country, and it is still operating.

We have in society a large number of persons who are borrowing funds every day to build homes for themselves, to buy pleasure cars, and to do many other things for which money is necessary. On the other hand, there are millions of persons who are saving money. They do not wish to hoard their money; they desire to invest it. The question naturally arises: How are the persons desiring funds brought into contact with the persons who are saving funds? Those desiring funds are brought into contact with those saving funds in three ways:

1. Direct contact may be established by one party going directly to another party and negotiating with him. A farmer borrowing from a wealthy townsman in order to secure funds to buy a farm, a merchant obtaining funds from a relative to start his store, and Bill asking John to lend him $10 until Saturday evening are everyday examples of this method.
2. A broker may become the active agent in getting the two parties together. He is the go-between and if the deal is made, he collects a commission for his service.
3. A financial institution, such as a bank, may be the means by which funds get shifted from the hands of the savers into the hands of the users. The present unit is concerned with an important financial institution—the bank. Our central objective is the ability to use banking services intelligently.

A bank is often thought of solely as an institution whose function is to safeguard money. One of its primary functions has always been the safeguarding of funds, but there are many other ways that it may serve society.

The businessman usually considers a bank as a medium through which he can pay his debts by means of checks. He also knows that if he is in need of financial aid and his credit is good, he can borrow from the bank.

The wage earner knows a bank as a place where he can cash his pay check and a place where his savings will be kept safely. Wealthy persons may think of a bank as an institution for the administration of estates and trust funds. To the producer a bank is a place where he may borrow funds required to manufacture or buy goods. Still others may think of a bank as an institution that issues paper money. All of these various views of the purposes of banks are, in general, correct. Money and credit are necessary in the operations of modern business. Therefore, banks are indispensable institutions in today's economic world.

c. Pretest

Sample Pretest Items

1. Define barter.
2. Why is money indispensable under the capitalistic system?
3. What is credit?
4. Describe the following: check, bank draft, bank statement.
5. Explain the significance of the following statement: "Savings deposits are not lawfully payable to any person other than the depositor himself."
6. Name and explain the common items found in the financial statement of a bank.
7. a. What are the duties of bank directors?
   b. State the duties of the following officers of a bank:
   (1) President; (2) Vice-President; (3) Cashier
8. What are the chief functions of Federal Reserve banks?
9. Discuss the various types of services that trust companies or the trust departments of commercial banks render for individuals and for corporations.
10. In what ways, from what sources, and for what purposes may farmers borrow?
11. a. What is mortgage banking?
    b. How does it differ in principle from savings banking and commercial banking?
12. What is a pass book and when is it used?
    What use should you make of the check stubs in your check book?
13. When a person wishes to open a checking account, what type of form does he fill out at the bank?
14. Name and explain the various types of indorsements.
15. If a check should be lost, what procedure would the loser follow?

2. *Objectives*[1]

a. Central Objective

The ability to use banking services intelligently

b. Contributory Objectives

(1) The ability to use banking business forms

(a) Checks
(b) Deposit slips
(c) Bank drafts
(d) Pass book
(e) Bank statement
(f) Certificate of deposit
(g) Signature card
(h) Promissory note

(2) Understanding how to manage personal finance problems

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[1] The objectives listed are merely illustrative. Changes will be made as a result of local pupil-teacher planning.

(3) Knowledge of the classification of banks with reference to types of services rendered

(4) Understanding the services rendered by banks to rural people

(5) Appreciation of the significant place that banks occupy in modern society

c. Indirect Objectives

(1) An increased interest in accuracy in all business matters

(2) Continuous effort to develop a well-modulated voice

(3) The ability to follow directions

This unit presupposes the fact that the students enrolled in the course have had courses in elementary economics and in junior business training

3. *Activities*

a. For Contributory Objectives (1) and (2) above

(1) Exercises: situations

(Work sheets will be used to give students experience in filling in forms such as checks and deposit slips)

(a) Using the forms for opening and maintaining a checking account; opening and maintaining a savings account; outlining the procedure for opening and using a checking account and a savings account.

(b) Each student whose allowance will permit will be encouraged to open a savings account at a local bank. Students who are responsible for their own bills will be encouraged to open a checking account.

(c) A bank for the campus is being planned. If the plans materialize, students in the class for whom this unit is being planned, will be active in the organization of the bank. If this cannot be done, efforts will be made to have a local bank set up a receiving and paying window on the campus. If either of these plans is realized, students will get experience in transferring their accounts from the local bank to the campus bank.

(2) Sample Check Test for Contributory Objectives (1) and (2)

Complete each of the sentences given by writing in the blank space one of the terms given at the left.

financial
interest
invest
per cent
principal
rate
thrift
wealth

1. If a person wishes his saving to increase, he must ______________ them so that they can earn interest.

2. If the people of a country do not practice ______________________ their total ____________________ will not increase.

3. When money is invested at the ______________ of 4 per cent, it is doubled in eighteen years.

(3) Problems

(a) Fill out the signature card for The Choral Club of Keystone College, whose checks are to be signed by James King, president, and Robert Lang, treasurer.

(b) Fill in the necessary information for a check: This check is for Henry Bronson in payment for 1,000 letterheads. Use the current date. The check is to be drawn on your personal checking account.

(c) By the 15th of this month, you are expected to make the regular deposit in your savings account.

(4) References: 6, 12, on page 136.

b. Contributory Objective No. (3) above—Knowledge of the classification of banks with reference to types of services rendered

(1) Exercises; situations:

(a) Members of the group will be sent to various banks in the city to secure illustrative materials.

(b) A librarian will be selected to take charge of the materials.

(c) Make a study of the banks in the city. Study bank statements (which may be secured from the local newspapers) from each bank as to the relative safeness of each one; find out which banks belong to the Federal Reserve System; find out if deposits are guaranteed. Discuss the merits and defects of each bank with your parents, friends, or business acquaintances to learn which they consider the soundest bank. From this study, make a list of standards for a sound bank to be used in a class discussion.

(2) Sample Check Test for Contributory Objective No. (3)

(a) Five classes of banks in the United States are: ______

(b) State banks and trust companies receive their aid from ______________________________

(c) Trust companies are organized primarily to accommodate ______________________________

(d) National banks must be members of the __________

(e) Five kinds of banks which are organized for limited purposes and which do not have checking accounts are: ______________________________

(f) Commercial banks are for the use of people engaged in ______________________________

(3) References: 1, 7, 9, 10 (page 136).

c. For Contributory Objective No. (4)—Understanding the services rendered by banks to rural people

(1) Exercises; projects

(a) The Farm Security Agent will be asked to cooperate in serving as a guide for the class while they make an inspection tour of a modern farm.

(b) The Federal Land Bank that serves the community where the institution is located is in Raleigh, North Carolina. A member of the class will write to an official in the Federal Land Bank, inviting him to the school to lead a discussion on the services of the Bank. Parents of some of the students (those who can make the trip; especially those who are farmers) will be invited to visit the class on this particular day.

(Note: Before the planned discussion, the class will have listed a series of questions to be used for the discussion.)

(2) Suggested Test: An essay on a topic that concerns rural financial problems.

d. For Contributory Objective No. (5)—Appreciation of the significant place that banks occupy in modern society

(1) Exercises; projects
    (a) Show a film on modern banking.
    (b) Class discussion (with instructor assisting with points that may cause confusion) on the financial statement of banks.

(2) Suggested test:

Have students examine current newspapers and magazines for articles dealing with present banking problems of national importance. From these articles a report will be prepared on the trends of modern banking.

(3) References: 1, 2, 11 (page 136).

4. *Suggested Test for Attainment of Central Objective*—The ability to use banking services intelligently.

PART I—TRUE AND FALSE TEST

Place a check mark (√) before each of the following statements which you think is true:

1. Checks are used as substitutes for money because it is unsafe to send actual money from place to place.
2. The person who writes the sight draft indorses it to his bank with a restrictive indorsement.
3. The depositor should sign his name on the signature card exactly as he intends to sign it on his checks.
4. The person receiving the bank draft may cash it without indorsement.
5. A promissory note can be sold.
6. A promissory note may bear interest.
7. A check is a bill of exchange.
8. Travelers' checks are used by travelers so that they will not have to carry with them large amounts of actual money.
9. A promissory note may be used to borrow money.
10. The drawee is the person who writes the sight draft.
11. A promissory note can be transferred only once.
12. The depositor writes the amount of his deposits in his pass book.
13. Savings banks handle checking accounts.
14. It is advisable for the person sending the bank draft to have the draft made out in his own favor.
15. A person who has a checking account can take his check book with him while he is traveling and use his personal checks.
16. The bank makes no charge for travelers' checks.
17. A businessman may use a sight draft to collect a bill due him.
18. The check stub should be filled before the check is written.
19. A bank draft is an order of a person on a bank.
20. The bank from which you obtain the travelers' check is known as the issuing bank.
21. The person receiving a bank draft in payment of a bill has to pay a collection fee.
22. The drawer is the person who is to pay the sight draft.
23. Banks charge their customers for bank drafts.
24. If a travelers' check is lost, the finder can get the money by signing his name on the check.

PART II—COMPLETION TEST

In the space to the right of the following statements, write the word or words that complete the statements:

1. When a person wishes to open a checking account, he goes to a bank and fills out a form known as a ____________.
2. When a depositor is making a deposit, he fills out a form called a ____________.
3. If the amount given in figures does not agree with the amount given in words, the amount that can be collected on the check is the amount written in ____________.
4. To prevent alterations in their checks, many businessmen stamp the amount into the paper with a machine called a ____________.
5. In listing checks on the deposit slip, the depositor sometimes does not use the name of the bank on which the check is drawn or the name of the city where the bank is located; instead of these he uses ____________.
6. To obtain a bank draft from a bank, it is necessary to write a personal check for the amount, and also to fill out a ____________.
7. The person who is sending the money writes on the back of the bank draft an ____________.
8. The person who is sending the money should have the bank draft written in favor of ____________.
9. The party who writes a promissory note is known as the ____________.
10. The amount which is written on a promissory note is known as the ____________.

PART III—RECALL TEST

Write a very brief answer to each of the following questions:

1. Why are checks used?
2. How would you go about opening a checking account in a bank?
3. If you owed $500 to Charles Wells, of Chicago, why would it be better to send Mr. Wells a bank draft rather than a check?
4. After Charles Wells has received the bank draft, how would he obtain the $500?
5. Suppose you are going on a trip to Europe. What will be the advantages of taking travelers' checks with you?
6. Suppose that you are in Geneva, Switzerland, and that you wish to cash one of your A.B.A. checks. Where can you cash the check, and what will you have to do?
7. Assume that you are a businessman, and that you wish to borrow $1,000 for sixty days on a promissory note. What will you have to do to obtain the money?
8. At the date of maturity of the promissory note, how will you pay back the money that you borrowed?
9. If you were a businessman for what two purposes would you use a sight draft?
10. Price & Holmes, of Chicago, owe you $250. You draw a sight draft on Price & Holmes. What do you have to do with this sight draft after you have written it in order to obtain the money from Price & Holmes?

PART IV—PROBLEMS (Necessary forms will be supplied)

1. You have in your safe the money and checks listed below, which you intend to deposit in the First National Bank today. Make out the deposit slip given on this page. (Items will be listed.)

. Assume that your checking account in the First National Bank shows a balance of $519.20. Between December 3 and December 9, you write the checks and make the deposits listed below. Make out the forms on this page, filling in both the check stubs and the checks. Start with 19 as the number of your first check. On each stub, figure the "Bal. car'd for'd," and then transfer this amount to the next check stub, as your "Bal. bro't for'd." (Items will be listed.)

3. Assume that your name is given as payee on three checks. You intend to have the checks cashed. Show what you must do to get the money on the checks.
4. Assume that you are a businessman, and that you own a store in Harrisburg, Pa. On September 12, you wish to borrow $500 from your bank, the Security National Bank, on a promissory note. The promissory note is payable thirty days after date, and its number is 25. Show how you make out this promissory note.
5. In a brief statement, tell what you would do if you were planning to purchase a farm but did not have sufficient cash to make the purchase.

5. *References for Learner*

1. Agger, E. E., *Money and Banking Today.* New York, Reynal and Hitchcock, 1941.
2. American Institute of Banking, *Farm Credit Administration.* New York, American Institute of Banking, 1934.
3. Beighey, C., and Spanabel, E. E., *Economic and Business Opportunities.* Philadelphia, The John C. Winston Co., 1938.
4. Coughlin, Rev. C. E., *Money; Questions and Answers.* Royal Oak, Michigan. National Union for Social Justice, 1936.
5. Gerstenberg, C., *Principles of Business.* New York, Prentice-Hall, Inc., 1946.
6. Jordan, D. F., *Managing Personal Finances.* New York, Prentice-Hall, Inc., 1946.
7. Lee, V. P., *Principles of Agricultural Credit.* New York, McGraw-Hill Book Company, Inc., 1930.
8. Odell, W. R., Clark, H. F. and others, *How Modern Business Serves Us.* Atlanta, Ginn & Co., 1937.
9. Peple, C. A. (Ed.), *Questions and Answers on the Federal Reserve System.* Richmond, Federal Reserve Bank of Richmond, October, 1926.
10. Prather, C. L., *Money and Banking.* Chicago, Richard D. Irwin, Inc., 1946.
11. Sparks, E. S., *History and Theory of Agricultural Credit in the United States.* New York, Thomas Y. Crowell Co., 1932.
12. Work books.
13. Current Literature.

6. *References for Teacher*

Ebersole, J. F., *Bank Management.* New York, McGraw-Hill Book Company, Inc., 1940.

Holdsworth, J. T., *Money and Banking.* New York, Appleton-Century-Crofts, Inc., 1923.

Keister, A. S., *Our Financial System.* New York, The Macmillan Company, 1930.

O'Connor, J. F. T., *Banks Under Roosevelt.* Chicago, Callaghan & Co., 1938.

Peterson, J. M., Cawthorne, D. R., Lohman, P. H., *Money and Banking.* New York, The Macmillan Company, 1941.

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Westerfield, R. B., *Money, Credit and Banking.* New York, The Ronald Press Co., 1938.

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Reports of Bank Commissioner

Reports of Farm Credit Administration

Current Literature

BASIC MATHEMATICS III

GRADE ELEVEN

Mathematics for the Citizen

Area of Activities

This phase of the mathematics program lays stress upon mathematics for the citizen, with subordinate emphasis on the individual, the learner, the consumer. There are many opportunities to correlate the work of this year with social studies courses.

One of the attributes of a good citizen is his ability to think clearly, to distinguish fact from fiction, to have an open mind. In the course parallel to this, Geometry, there is development of an approach to teaching critical thinking (page 219). Teachers in Basic Mathematics may select ideas from that outline and should be on the watch for opportunities to apply types of reasoning to the problems which arise in this year's work on community situations, such as the study of taxes, the analysis of advertisements, the planning of community facilities and activities. In the area of reasoning, help should be given in the following aspects: definitions, indirect method, analogy, circuitous reasoning, generalizations.

Much of the work in making a better community and in carrying on community work has its roots in geometry. Design, measurement, ratio, and proportion are only a few of the geometric understandings involved. By referring to the topics in Geometry marked *(Basic)* (page 213), the teacher will find help in planning class activities. Use should be made also of the *Eighteenth Yearbook of the National Council of Teachers of Mathematics.*

The following areas of pupil experiences are suggested:

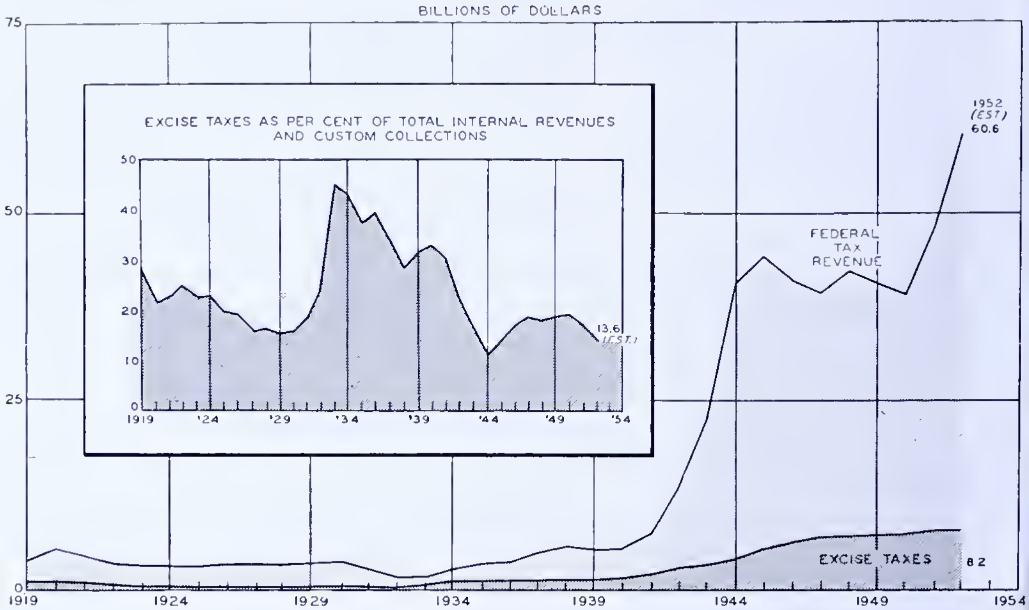
I. PAYING FOR GOVERNMENT SERVICES BY TAXATION—See Unit II, page 144.

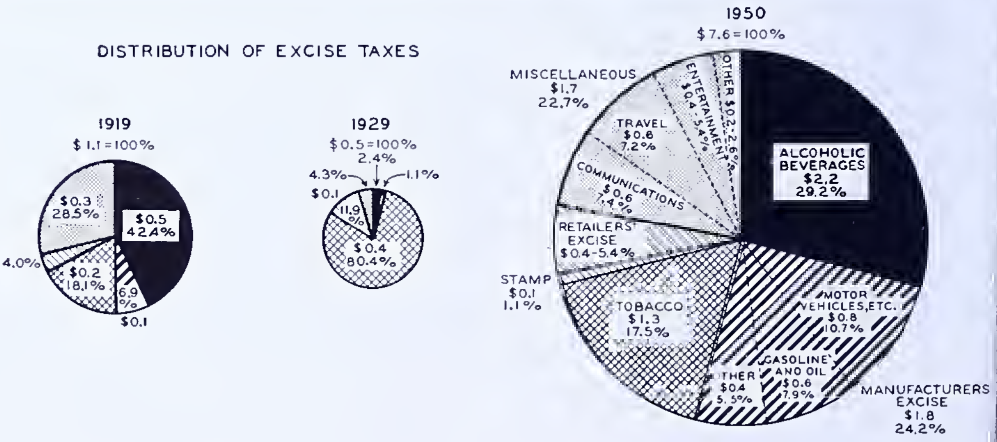
A. *Introductory*

A key problem is the developing of wholesome attitudes toward government expenditures. Questions to start investigation might include:

1. Why are taxes so high?
2. What is a 15-mill tax limitation?
3. Should we have a sales tax?
4. Should we have an income tax?
5. Why do different states have different taxes?
6. Why doesn't a dollar always buy the same amount of goods?

FEDERAL EXCISE TAXES
UNITED STATES, FISCAL YEARS 1919-1952





Revenue raised by federal excise taxes has grown considerably since 1919. This kind of tax, principally on liquor and tobacco, accounted for a little more than one billion dollars in 1919. In fiscal 1952, the more diversified group of existing excise taxes will produce an estimated $8 billion. In relation to total taxes, however, they constitute a smaller share now than in the earlier year.

SOURCE: Treasury Department

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B. *Possible topics:*

1. Spending the tax dollar wisely
2. Coining money and regulating its use
3. Providing for prompt payment of all obligations of taxation
4. Understanding the purposes and machinery of taxation—local, state, federal
5. Comparing taxes paid by Americans and those paid by people living in other countries
4. Study of local education costs

C. *Visual aids:*

Federal Taxation. 1 reel. B and W or Color, also filmstrip. Coronet

Property Taxation. 1 reel. Encyclopaedia Britannica

D. *References:*

Benz, Harry E., *Arithmetic in General Education.* Chapter VI, "Arithmetic in the Senior High School." Sixteenth Yearbook of the National Council of Teachers of Mathematics. New York, Bureau of Publications, Teachers College, Columbia University, pp. 123-4

Krug, Edward A., *Why Taxes? What They Buy for Us.* For the Committee on Experimental Units of the North Central Association of Colleges and Secondary Schools. Ginn and Company, Boston

NEA Department of Supervision and Curriculum Development, *Taxes.* 31 pages. 30 cents. Published by Building America, 2 West 45th St., New York 19, N. Y. Well illustrated and excellent for the classroom reading table

There should also be available:

A good textbook on economics, giving over-all view of taxation
A copy of last year's federal income tax blank with instructions

II. ASSURING COMMUNITY WELFARE BY PROVIDING FOR PERSONAL AND FAMILY SECURITY—INSURANCE

While insurance is considered in other grades, the point of view and the nature of the work should be different each year as the pupils mature so that an understanding of these topics develops in a spiral effect.

A. *Health*

What does the community do to safeguard health? What responsibilities has the individual? (Study sanitation department and water department budgets.) How does personal insurance supplement this work and add to community security?

B. *Accident*

Study relationship between accident rate and insurance rates. How can both individuals and communities counteract the threat of accidents?[1]

C. *Fire*

Unit I, page 141, considers the threat of fires and fire insurance.

III. PLANNING FOR SAVINGS AND INVESTMENTS

A. Investment in town or city bonds

B. Opening and maintaining a savings account for later use in property improvement

IV. HELPING TO SOLVE COMMUNITY PROBLEMS

A. *Establishment and maintenance of recreational facilities*

1. Make a survey of needs for leisure time activities.
    a. Collect numerical data.
    b. Make maps of playground areas.
2. Note calculations necessary in working on problems involving: necessary space, cost of projects, number of individuals involved, cost of maintaining such facilities, records o receipts in situations where dues are required.

B. *Better communities through better housing*

1. Planning a housing project
    a. Layout of the area
    Uses of geometric formulas, principles, instruments scale drawing, indirect measurement
    b. Determination of cost of building or remodeling
    c. Financing the project
    d. Determination of rental or sale prices
2. Home construction and improvements
    a. Study how geometric forms are used in architecture, i the kitchen, and in other instances in home building.
    b. Display a series of pictures of various types of building which exemplify the simple solid geometric forms.
    c. Discuss such questions as:
        (1) Why are rooms made in the shape of rectangula solids and not spheres?

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[1] See "Safety Hints" in such textbooks as *Today's Geometry,* by Reichgott and Spiller. Se Geometry outline for uses of geometry in traffic-control planning, page 214.

      (2) Why is the barn (or roof) gable triangular?
      (3) Why are the bridge structures spanning streams near the project made up of triangles?
    d. In connection with a study of design, introduce point symmetry and line symmetry.
  3. City planning—scale drawings

C. *Community problems of transportation and communication*
  1. Automobile regulations
    a. Registration data and fees
    b. Traffic regulations
    c. Cost and extent of city (or town) -owned vehicles
    d. Parking facilities
      Planning for various types
      Parking meters—initial cost, maintenance, income
  2. Air travel
    a. Cost of maintaining municipal airports
    b. "Breakdown" of costs of operating an airline
    c. Determination of costs to the traveler
  3. Telephone and telegraph
    a. Cost versus benefits
    b. City plans to remove overhead wires
  4. Maintenance of streets and roads
    Costs of various types of road surfaces

ILLUSTRATIVE UNITS

UNIT I

A STUDY OF FIRE INSURANCE

1. *Preliminary Teacher Planning*
  a. The problem should originate through a feeling that the future citizens of our democracy will in a short time be buying many different kinds of insurance, especially fire insurance. Millions of dollars' worth of property is lost every year because of fire. In many cases fire is due, not to any fault of the property owner, but to such things as carelessness on the part of others, defective wiring, lightning, etc. These are often beyond the owner's control.
  b. Some of the understandings which should be sought are:
    (1) A good way to save money is to take out fire insurance.

(2) Rates of insurance depend upon construction, location, and use made of the building.

(3) Fire insurance is the sharing of risks; it is not gambling.

(4) Thousands of persons in the United States lose their homes each year because of fire.

(5) The annual property loss due to fire amounts to nearly three hundred million dollars.

(6) Cooperative effort is concern for others as well as for self.

(7) Working together and comparing steps in insurance work provide practice in democratic processes.

c. The study may be presented by the teacher's telling some facts pertaining to the origin of insurance. In the early days of trade and commerce the risks of travel were so great that caravans and merchant ships were often lost. As a result, in the course of time, companies were formed which protected merchants against losses upon payment of certain fees. Out of such practice the large insurance companies gradually developed. Today insurance is one of our biggest businesses, employing many persons at good salaries. No doubt some pupils in this class will take up insurance as a livelihood.

d. The class selects a general chairman and committee chairmen. The class then organizes itself into two general units. Each pupil chooses a problem with which he desires to work.

(1) One problem might be a report on the following:
  a. News clippings from newspapers on fire losses
  b. Radio accounts of recent fires and their losses

(2) Another problem might be to secure information pertaining to major fires occurring in your community in the past twenty-five years.

2. *Orientation*

a. An overview conducted by pupils:

Rates of insurance depend upon

(1) Location of the property

(2) Use made of the building

(3) Construction of the building

(4) Use and construction of the neighboring buildings

b. Discussion and reports on causes of destructive fires. Where possible secure data suitable for making charts or graphs on the frequency of causes.
   (1) Matches
   (2) Smoking
   (3) Spontaneous combustion
   (4) Defective chimneys and flues
   (5) Overheated stoves and boilers
   (6) Defective wiring
   (7) Lightning
   (8) Sparks on roofs
   (9) Open fireplaces
   (10) Rubbish
   (11) Ignition of hot grease
   (12) Fireworks, etc.

c. Collection by pupils of valuable information for committee work and class discussion, such as:
   (1) Insurance is a cooperative arrangement.
   (2) Many homes are destroyed each year by fire; therefore property owners should protect their homes with insurance.

3. *Learning Activities*

Committee work (pupil participation)

a. Secure information pertaining to rates on insurance by interviewing local insurance company representatives.

b. Compile a true-false test for later use on material acquired by committee work.

c. Through committee work learn the following:
   (1) Whether the pupils' parents have fire insurance on their homes and furniture.
   (2) Whose responsibility is insurance against fire—the person renting a house or the person owning the building?

4. *Culminating Activities*

a. Pupils bring in circulars which give information concerning various forms of insurance.

b. Cancelled insurance policies might be examined and studied.

c. Reports could be given in class on valuable data received from their parents on insurance.

d. Through group reports and discussions the pupils develop an understanding of the meaning of such terms as: policy, face of the policy, premium, rate of insurance, etc.

UNIT II

WHAT DO WE GET FOR OUR TAXES?

1. *Overview by Teacher and Planning with Pupils*
2. *Pupils Concerns*
    a. What does "take home pay" mean?
    b. Who pays for school?
    c. Are we getting too many or too few services for our tax money?
    d. Will taxes always be getting higher?
    e. Is there a chance for a reduction in taxes?
    f. Is our tax money being put to good use?
3. *Objectives*
    a. To acquaint ourselves with our system of taxation in a democrac
    b. To become better informed citizens as to how our tax money i spent or should be spent
    c. To spread this knowledge among less informed people
    d. To awaken interest in our city and nation's tax problems
4. *Content Outline*
    a. The government's need for money
    b. Kinds of taxes
        (1) Good taxes
        (2) Levying and collecting taxes
    c. Services to us: Taxes vs. services
    d. Tax reforms
5. *Activities*
    a. Films:
        Property Taxation
        Civil Service Advancing
        Defending the Cities' Health
        Federal Taxation

b. Survey—All activities in which the class participated that were taxable (motion pictures, sodas, skating, etc.). Make chart tabulating amounts spent for taxes.

c. Posters of all different types of taxes. Receipts for water tax, real estate, amusement, etc.

d. Newspaper discussions on tax affairs

e. Scrapbooks

f. Neighborhood survey of things that tax money provides (police, lights, fire protection, etc.)

g. Interview the City Treasurer on:
   (1) Where the tax dollar comes from
   (2) Where the tax dollar goes

h. Investigate the Federal budget. Where does the money come from?

i. Calculate the present income tax for $5,000.00, $10,000.00, etc.

j. Investigate cost of education
   (1) How much does the State provide?
   (2) How much does the community provide?
   (3) Should the schools have more?
   (4) What is the cost per pupil?

. *Culminating Activity*

a. Class evaluation of how much we get daily for our taxes
b. Panel discussion on whether we get enough for our money

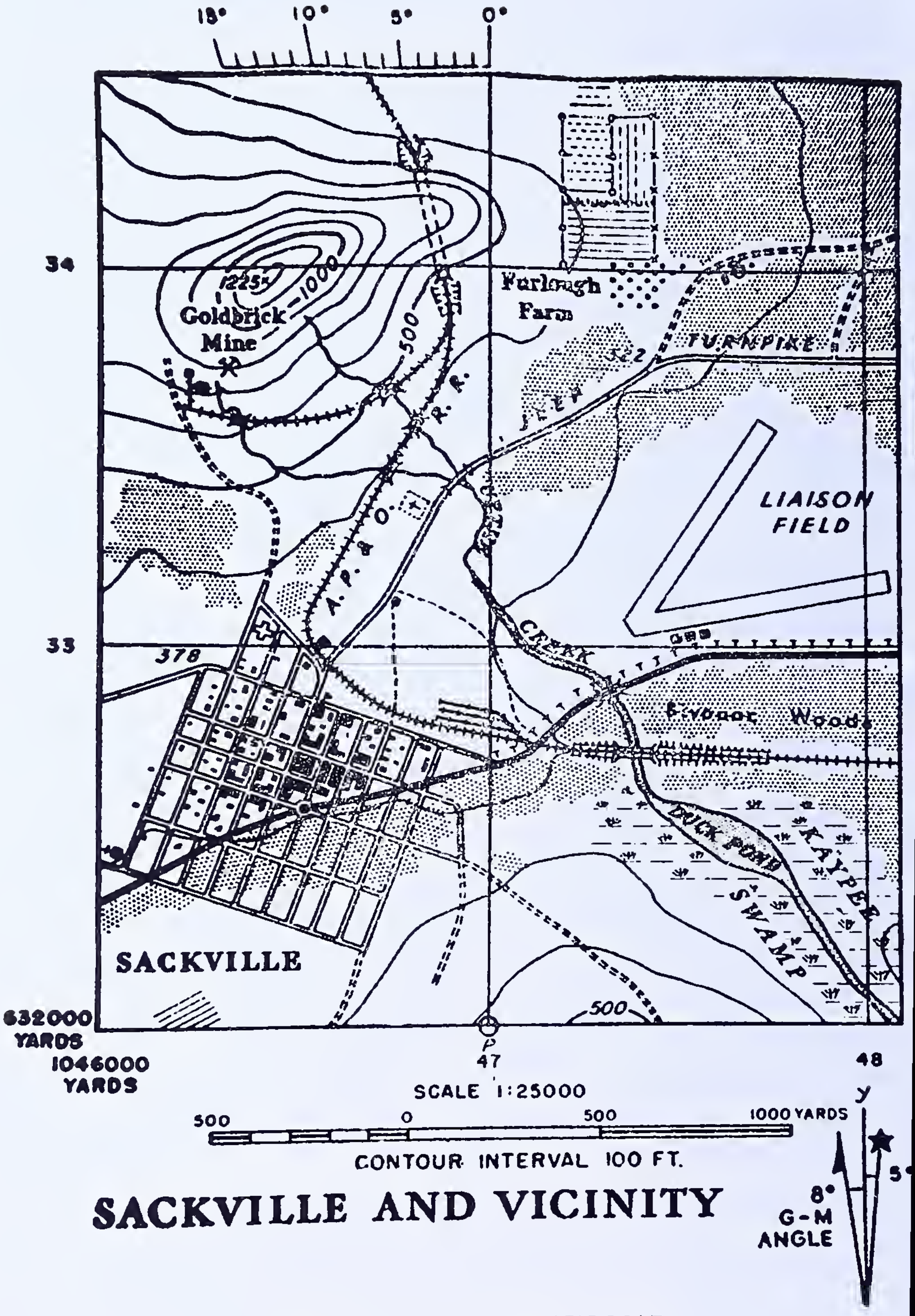
. *References*

Books used by Pupils:

a. *Building America—Taxation*
b. *Taxes and What They Buy*
c. *Our Economic and Community Life*
d. *Building Citizenship*—Hughes

Teacher References:

a. *A Primer of Economics*
b. *The State Sales Tax*—Nicholas
c. *The Principles of Economics*—Faulkner
d. *Building Citizenship*—Hughes



SECTION OF A TERRAIN MAP

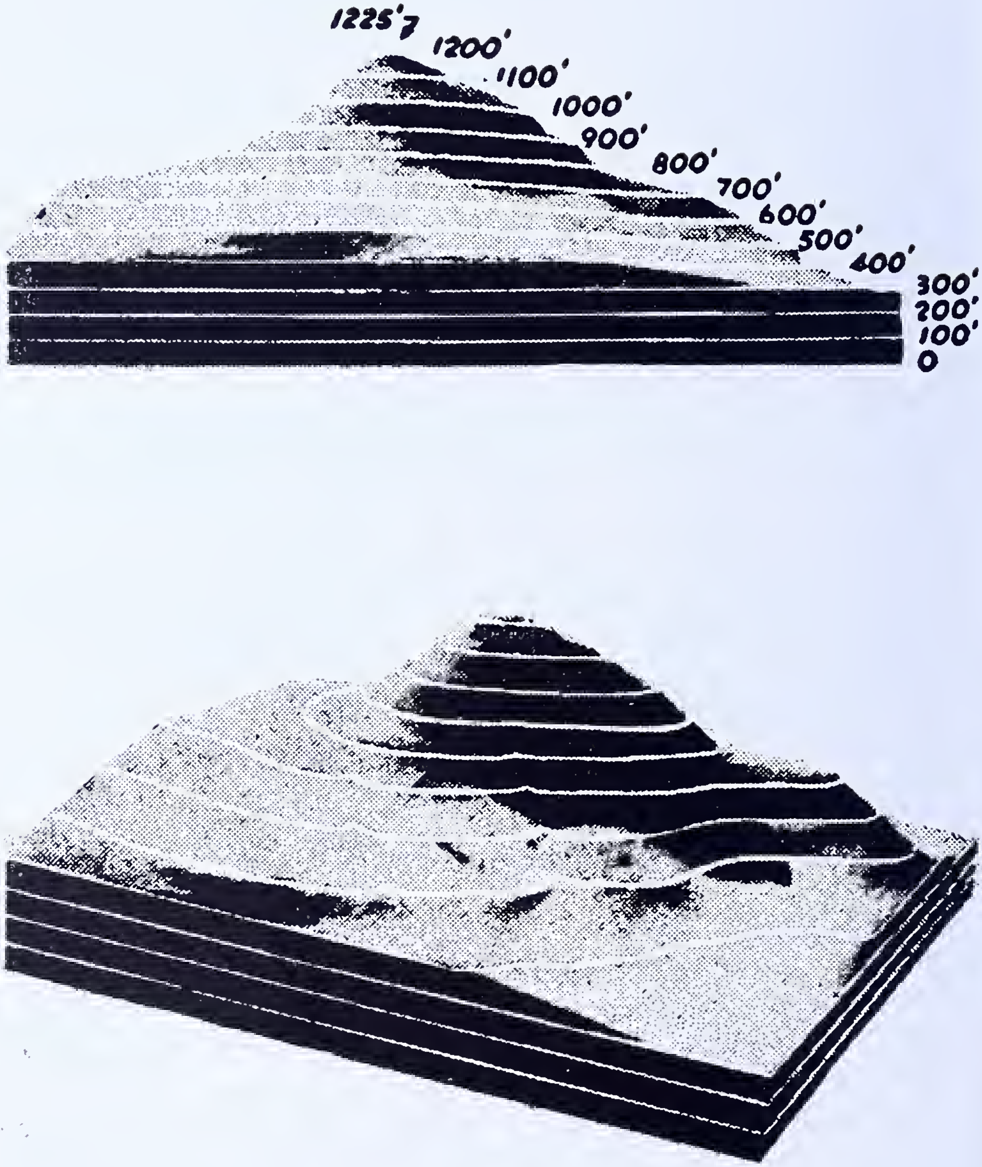
UNIT III

ELEMENTARY MAP AND AERIAL PHOTOGRAPHY READING

1. *Overview by Teacher and Planning with Pupils*
   a. General discussion on maps and their use
   b. Examination of sample maps and aerial photographs
   c. Viewing of films on map reading

2. *Objective—Requirements for Interpreting Maps Accurately*
   a. Knowledge of geographical features
   b. Knowledge of ways in which elevation is shown and measured
   c. Ability to measure distance and direction
   d. Ability to locate points by coordinates
   e. Ability to orient a map with the ground

3. *Sample Learning Activities* (Class, Committee, and Individual Work with Reports)
   a. Signs and Symbols
      (1) Significance of the legend—name of map, graphic scale, representative fraction, contour interval
      (2) Significance of colors used on a ground map
         (a) Works of man
         (b) Vegetation
         (c) Elevation
         (d) Water
      (3) Colors used to designate roads of various types
      (4) Geographical features and their names: ridge, saddle, draw, etc.
   b. Location of Points
      (1) Rule for reading coordinates (read right up)
      (2) Significance of grid lines (printed or drawn by pupil)
      (3) Conventional distances between grid lines on maps of different scales
      (4) Preparation and use of a grid card (dividing grid distances into tenths) and practice in its use and in writing coordinates (10.7–6.5)
      (5) Report on the zero lines, Greenwich meridian and the equator, and on parallels of latitude and longitude
   c. Measurement of Distances
      (1) Significance of the representative fraction and the meaning of the scale



CONTOUR ELEVATIONS

  (2) Use of a ruler, a strip of paper and a pin, the dividers, and a map measurer in measuring distances
  (3) Use of mileage figures and charts on road maps

d. Measurement of Direction
  (1) Use of a compass and its characteristics. Discuss its purpose
  (2) Meaning of the terms: magnetic north, true north, azimuth, declination, North Star, back azimuth
  (3) Use of a protractor and grids in measuring azimuths on a map
  (4) Corrections for declination. Use of LARD formula (Left, add; Right, deduct)
  (5) Outdoor practice in the use of the compass
  (6) Practice in orienting maps by compass and by observation
  (7) Finding North by using a watch

e. Measurement of elevation
  (1) Study of the use of contour lines in measuring elevation
  (2) Significance of contour lines that are close together or far apart or form *V's* when crossing streams
  (3) Meaning of the expressions "contour interval" and "vertical interval"
  (4) Use of sea level in measuring elevation
  (5) Practice in estimating visibility from point to point on a map and in constructing a profile
  (6) Practice in accentuating terrain features by drawing in stream lines in blue and ridge lines in brown or some other colors

f. Reading aerial photographs
  (1) Position of shadows in reading
  (2) Factors which help to distinguish objects (size, shape, texture, shadow)
  (3) Advantages of an aerial photograph over a topographical map
  (4) Significance and meaning of grid lines on an aerial photograph
  (5) Practice in orienting an aerial photograph

(6) Significance of numbers (such as 12:28 : 1330) printed on the margin

(7) Determination of the scale of an aerial photograph

g. Map-making class activity

(1) Construction of stride scale

(2) Construction of a slope board

(3) Making a plane table survey on surrounding terrain

(4) Practice in plotting contours, using slope board

4. *Culminating Activities*

(1) Demonstration of prepared maps, stride scales, etc.

(2) Committee reports and demonstrations

(3) Class discussion on achievement of objectives and effectiveness of work

(4) Use of self-appraisal check list

(5) Review of films

(6) Achievement test and drill where need is shown

5. *Bibliography*

FM 21-25,[1]	Elementary Map and Aerial Photograph Reading	
FM 21-26,[1]	Advanced Map and Aerial Photograph Reading	
FM 21-30,[1]	Conventional Signs, Military Symbols and Abbreviations	
FM 21-35,[1]	Sketching	
FM 30-21,[1]	Aerial Photography, Military Application	
TF D21-2071,[2]	Basic Map Reading, Part I	—Conventional Signs
TF D21-2072,[2]	Basic Map Reading, Part II	—Elevation, Distance, and Grid
TF D21-2073,[2]	Basic Map Reading, Part III	—Direction, Orientation, and L cation without compass
TF D21-2074,[2]	Basic Map Reading, Part IV	—Direction, Orientation, and L cation without compass
TF D21-2075,[2]	Basic Map Reading, Part V	—Photos and Photomaps

NOTE: Much of the work can be done on an ordinary road map. Simple ground maps may secured from the Military Service Publishing Company, Harrisburg, Pa.
FM: Field Manual
TF: Training Film

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[1] Secure through local recruiting officer, military post, or from the Superintendent of Document U. S. Government Printing Office, Washington, D. C.

[2] Secure through local recruiting officer or military post.

PERFORMANCE TEST

For Use With Individual Pupils

NOTE: The teacher should have available: ground map, an air photo, ruler, map measurer or strip of paper with a pin, compass and protractor. Forms may be mimeographed for use with individual pupils, or for pupil self-appraisal.

		CHECK	
1. *Meaning of Items on the Legend*	*Yes*	*?*	*No*
a. What is the representative fraction?			
b. How many inches on the ground is represented by an inch on the map?			
c. What is the contour interval?			
d. What is the magnetic declination?			
e. What is the grid declination?			

		CHECK	
2. *Meaning of Topographical Symbols* (Point to each on a map)	*Yes*	*?*	*No*
a. What does this represent? (Improved road)			
b. What does this represent? (Trail)			
c. What does this represent? (Contour line)			
d. What does this represent? (Bridge)			
e. What do these represent? (Buildings)			

		CHECK	
3. *Location of Points* (Points marked on map, find coordinates)	*Yes*	*?*	*No*
a. V			
b. W			
c. X			
d. Y			
e. Z			

		CHECK	
4. *Measurement of Distance* Require each pupil to use correctly: a ruler, edge of paper, or map measurer and apply it to the graphic scale.	*Yes*	*?*	*No*
a. What is the straight line distance from X to Y?			
b. What is the straight line distance from A to B?			
c. What is the road distance from X to Y?			
d. What is the road distance from A to B?			
e. If the distance between two points on this map is six inches, how many miles is it on the ground?			

		CHECK	
5. *Measurement of Direction*	*Yes*	*?*	*No*
a. What is the grid azimuth from X to Y? (Pupil reads azimuth with protractor.)			
b. If you were at X on the ground and wanted to go on a straight line to Y, what compass reading would you follow? (Pupil changes grid azimuth in item a to a magnetic azimuth.)			
c. If you were at Y and wanted to go to X, what azimuth would you follow? (Pupil figures back azimuth—add or subtract 180°.)			
d. If you read an azimuth of 40° between two points on the ground with a compass, what azimuth would you plot on this map with a protractor? (Pupil changes magnetic azimuth to a grid azimuth.)			
e. Orient this map, using this compass. (Pupil orients the map on the floor, placing compass needle over magnetic arrow on the map and turning the map.)			

6. *Elevations*

		CHECK	
(Points marked on map)	*Yes*	*?*	*N*
Give the elevation of the following points.			
a. V			
b. W			
c. X			
d. Y			
e. Z			

		CHECK	
7. *Use of Air Photo*	*Yes*	*?*	*N*
a. Show how to hold the air photo to study it. (Shadows should be toward the pupil)			
b. Explain the meaning of the printing on the photo. (Location, date, hour, scale—or focal length and elevation, direction arrow, and file number. All may not be present.)			
c. How is the scale of an air photo determined? (The ratio between the focal length of the camera, in inches, to the elevation above the ground in inches.)			
d. Identify the following objects: (Point)			
(1) House			
(2) Road			
(3) Stream			
(4) Hill, etc.			
e. If the scale were not marked on the photo and you were on the ground, how could you determine it? Answer: By pacing the distance between two visible points and calculating the scale.			

UNIT IV

ORIENTATION UNIT
ELEMENTARY NAVIGATION[1]

1. *Introduction*

It is hoped that this unit will arouse the interest of the students to the point where they are looking forward to taking a navigation course, and that they will have more critical understanding of strategic geographical locations.

2. *Objectives*

a. Central Objective
Understanding the background material for navigation

b. Contributory Objectives
(1) Knowledge of the prime meridian, the equator, and the earth's poles
(2) Knowledge of latitude
(3) Knowledge of longitude
(4) Understanding of the time zones
(5) Understanding of map distortions and shortest routes

c. Indirect Objectives
(1) Interest in the study of navigation
(2) Increased vocabulary of navigational terms
(3) Interest in geography

3. *Presentation and Exploratory Questions*

As we recall from our history, Magellan's ship was the first to circumnavigate the globe. This feat was accomplished without charts or any of the refined navigational equipment that we have today. Magellan's only guide was a magnetic compass, and his trip was a sort of hit-or-miss venture. Today, however, one can go to any section of the world by either regularly scheduled airplane or ocean liner trips; and he will have constant knowledge of his position on the earth's surface throughout the flight or cruise. Navigation is developing more and more into an electronic operation, but every navigator must know the fundamental divisions of the earth's surface and how to locate positions in terms of these divisions.

(1) Which city is farther south—Rome or Philadelphia?
(2) When it is noon in London what time is it in New York City?
(3) Which is the farther west—Chile or Florida?

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[1] School of Education, University of Pennsylvania.

4. *Sample Learning Activities*

a. For the first contributory objective—Knowledge of the prime meridian, the equator, and the earth's poles

The earth rotates about an axis, an imaginary line through its center. The ends of this axis are called the poles, North Pole and South Pole. An imaginary circle around the earth, halfway between the poles, is called the equator. An imaginary circle through the poles and through Greenwich, England, is called the prime meridian.

Locate the poles, equator, and prime meridian on a mounted globe.

b. For the second contributory objective—Knowledge of latitude.

The latitude of a place is its distance north or south of the equator, measured in degrees. The circles of latitude drawn on the globe every fifteen degrees north and south from the equator are known as parallels of latitude. Since they are parallel, any two of them are everywhere equidistant from each other. The equator is the line of zero latitude. Points on parallels to the north of the equator are designated as north latitude; those to the south are called south latitude. The degrees of latitude are subdivided into minutes and seconds for accurate navigation—each minute of latitude equals one nautical mile in distance. The navigator's symbol for latitude is "Lat."

Locate and label correctly the latitude of Rome, Philadelphia, Rio de Janeiro, and Capetown.

c. For the third contributory objective—Knowledge of longitude.

The longitude of a place is its distance east or west of the prime meridian, measured in degrees. It is measured from Greenwich eastward 180 degrees and westward 180 degrees. Lines of longitude are called meridians. The 180th meridian is known as the *international date line.* The symbol for longitude is "Long."

Locate the international date line on the globe and name a group of islands through which it passes.

If you know the latitude and longitude of any position you can readily locate it on a chart or a globe. It is the navigator's job to know his position at all times.

Locate and label a group of islands at approximately 162 degrees west longitude and 22 degrees north latitude.

d. For the fourth contributory objective—Understanding the time zones.

When it is midnight in Philadelphia it is 5 A. M. of the following day in London. The earth is divided into time zones each being 15 degrees of longitude, measured seven and one-half degrees on either side of the meridians divisible by 15. Each time zone is equivalent to one hour. The zones are labeled plus one, plus two, etc., westward from the Greenwich zone, and minus one, minus two, etc., eastward from the Greenwich zone. Since Philadelphia is in the plus five zone (near the 75th meridian) one adds five hours to Philadelphia time to get London time, London being in the Greenwich zone. The plus and minus twelve zones meet at the international date line and are actually one-half zones. When one crosses the 180th meridian going westward he skips twenty-four hours or one day on his calendar since he passes from the plus twelve zone to the minus twelve zone. Here is the only place one can completely lose a day of his life (going westward) and can live the same day twice (going eastward). The international date line is the original point of date change; the date change moves progressively westward at one hour intervals for each zone.

The navigator must have a thorough knowledge of time zones and changes because his computations are all based on Greenwich time.

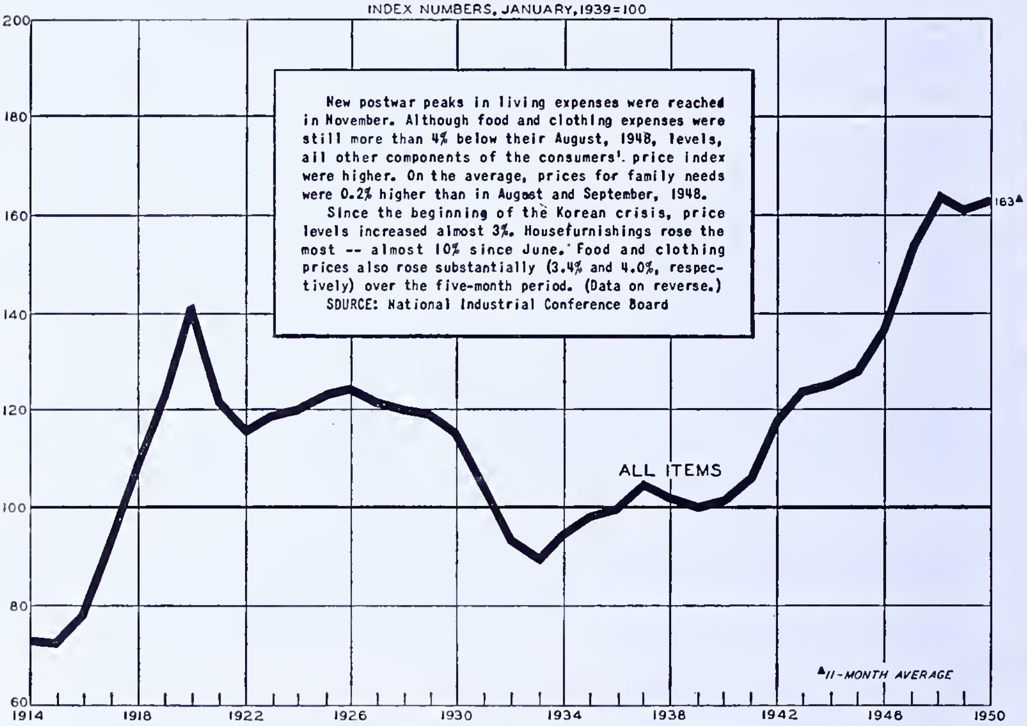
Locate Chicago, Honolulu, Shanghai, and Cairo on the globe and report their time zones.

c. For the fifth contributory objective—Map distortions and shortest routes

The earth is spherical in shape, and so the only way to chart its surface without distortion is to make a globe. You probably know from experience that you can't take a half section of a hollow rubber ball and flatten it.

For practical purposes, however, it is necessary to represent the earth's surface on charts. The most common projection of the earth is the Mercator chart. This is the type reproduced on most wall charts and in books and also used by the surface navigator. Most people believe that a straight line connecting two points on such a chart is the shortest route. This is incorrect and can be proved so by measuring, with a piece of string, on a globe the distance between Los Angeles and London, for example. The string measurement on the globe is the shortest route. Note six or eight check points along the path of the string and then plot these on a Mercator chart. It will be obvious that they do not lie in a straight line.

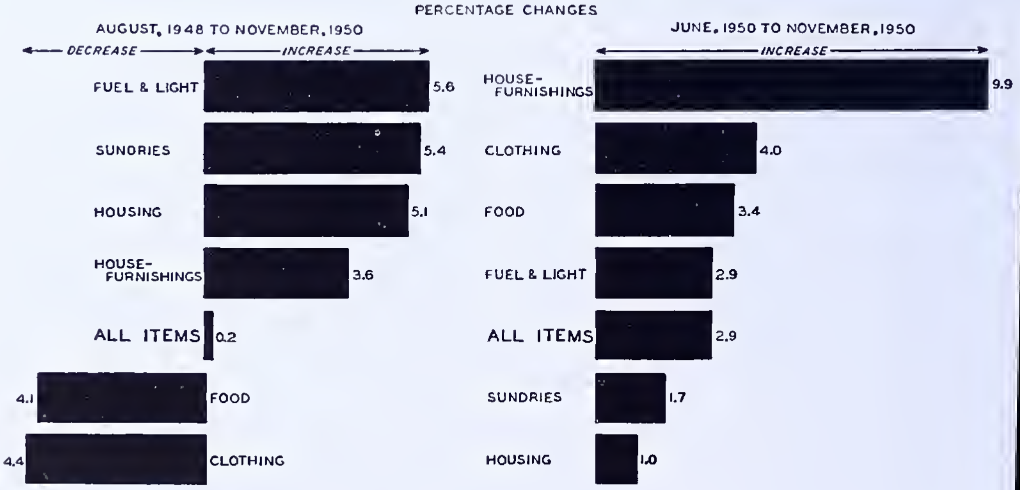
CONSUMERS' PRICE INDEX
UNITED STATES, 1914-1950



New postwar peaks in living expenses were reached in November. Although food and clothing expenses were still more than 4% below their August, 1948, levels, all other components of the consumers' price index were higher. On the average, prices for family needs were 0.2% higher than in August and September, 1948.

Since the beginning of the Korean crisis, price levels increased almost 3%. Housefurnishings rose the most -- almost 10% since June. Food and clothing prices also rose substantially (3.4% and 4.0%, respectively) over the five-month period. (Data on reverse.)

SOURCE: National Industrial Conference Board



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THE CONFERENCE BOARD

The polar projection chart is used for air routes. On this chart one can pick out shortest (or "great circle") routes more easily. Such charts can reproduce one hemisphere without much distortion, i.e., the hemisphere which contains the pole used as center of the chart. Compare Mercator and polar projection charts with a globe.

5. *Suggested Evaluation*

A written objective test including map identification items

BASIC MATHEMATICS IV

GRADE TWELVE

Mathematics for the Consumer

Suggested Topics

Pupils in the last year of the high school are at a crucial stage. For many, especially those pursuing the Basic Mathematics courses, the twelfth grade will mark the end of formal education. Therefore attention must be given to preparing for the problems of living which will be more realistic once the pupil is on his own. It is also a time to look backward in order to survey the most significant learnings which have developed during the secondary school years.

During this time the basic mathematics program has been directed toward the mathematical aspects of many of the problems of life. Some of these need to be reviewed and extended in terms of the greater maturity and understanding of the individuals.

It is the aim of the twelfth year in mathematics to meet such challenges. *This phase of the program, then, lays stress upon mathematics for the consumer.* The problem can be broken roughly into two areas—(1) acquiring the family income, (2) using the family income wisely. It will be noted that the first of these recalls Basic Mathematics II, Mathematics for the Earner. The emphasis in Basic Mathematics IV is less on the exploratory or guidance purpose and more on income as a family problem. Accompanying this outline is a resource unit, Family Financial Security. The topics and activities will suggest many possibilities for teachers working with high school seniors.

Other suggested topics follow:

1. *Establishing and Maintaining a Home*—"I want to build a house."
   a. Business involved in the building of a home
   In what ways may a buyer's ignorance cost him money in the building of a home?
   b. The site
      (1) How is mathematics used in measuring? What numerical

trigonometry is basic to necessary surveying? Road grades leading to property are measured by trigonometry.

(2) Study the cost of land as it is affected by location, size, improvements, development.
What do recent housing statistics show to be the housing conditions in both cities and rural areas? (Note: Here is an excellent place to introduce a brief study of elementary statistics. See the outline for Specialized Mathematics, Twelfth Year, page 194, for suggested topics and sources.)

(3) Investigate the statement: "The housing industry is backward in its building methods. Houses are constructed in much the same manner as 100 years ago, and the same types of materials are used. It has been estimated that 53 per cent of the cost of construction represents waste of both labor and materials."

(4) Questions of concern to the teacher:
  (a) How can the training in critical thinking apply here?
  (b) What can be done to improve the situation by more effective use of mathematics?

c. Make a comparison of building a home with (1) the cost of buying, (2) the cost of renting. What factors other than cost should affect an informed decision? How has the cost of homes changed in comparison with the cost of items such as automobiles and radios in the past twenty years? What factors affect cost?

d. Cost of furnishing a home

(1) Investigate consumer studies on how to buy items intelligently.[1]
  (a) What quantitative concepts combine with buymanship? For example, a principle for determining size of refrigerator concerns the number in family (allow 1.2 cubic feet per member of family). Here is a good chance to stress the use of common sense in buying according to "formulas"—i.e., a family of two would not try to get a refrigerator with 2.4 cubic feet.
  (b) Individual pupil investigation of some home furnishing item for special report
  (c) Use of geometry in making designs

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[1] *Consumers' Research Bulletin* has many reports of a mathematical nature which serve as guides to a "good buy."

    (2) Measure at home to determine cost of purchasing wall paper, rugs, paint, etc.

  e. Cost of upkeep—usual rates of deterioration

  f. Financing the home
    (1) How does one determine the price a family can afford?
    (2) How are mortgages secured?
    (3) Investigate and make reports on items such as
      (a) Loans on real estate, interest and costs
      (b) Costs of fire and other dwelling hazard insurance
      (c) Local property taxes

  g. Summary project:
  Assume an annual family income of $3,000 (or whatever is an "average" income in the local community). Select a house plan for a family of four—two adults, two children; determine the type of construction, total costs (including the land), methods of financing.

2. *Budgeting on a Family Basis.* (See Family Financial Security Unit, page 163.)
  a. Prepare a budget for the family in 1, g—What percentage is usually allotted to various items in model budgets?
  b. Study the effect of a salary increase upon
    (1) The percentage of savings
    (2) The percentage of expenditures for food

3. *Insurance.* (See Family Security Unit, page 163.)

4. *Investing*

Study market reports—make graphs of changes; compare trends.

5. *Improving Shopping and Buying Methods*
  a. Informed buying
    (1) Watch for current statements (such as appear weekly in the Home Section of many metropolitan daily papers) like the following:

"Agriculture Department specialists advise housewives to check prices of the smaller eggs for better values on a pound-for-pound basis. A price table shows that if large eggs are selling at 71 to 75 cents a dozen, the small ones are as good or a better value at 53 to 56 cents. And at 76 to 80 a dozen for large you're getting a good buy in small ones at 57 to 60 cents," according to the Production and Marketing Administration. "The size of an egg has nothing to do with its quality."

(2) Analysis of advertising—further development of critical thinking. Practice interpretation of data presented graphically.

(3) Understanding of statistics as used in daily reading—(See Specialized Mathematics, Grade 12, page 239.)

b. What are the advantages and disadvantages of installment buying?

(1) Study interest charges to see what the real rate is.

(2) What is the per cent of increase of cost when buying "on time"?

c. Study buying information on foods that will serve to stretch the family dollar.

(1) Using similar products, show how buymanship considers factors such as grade, quality, quantity, and price.

(2) A box of prunes reads "60-70's." What does that mean to the buyer?

d. How do charge accounts affect family buying?
Investigate local practices by contacting credit managers.

6. *Credit and Lending Services*

a. How does one establish a good credit rating?

b. Study of loan agencies. What are the financial plans with

(1) Banks?
(2) Finance companies?
(3) Pawnbrokers?
(4) Credit unions?

See Unit I, page 161.

7. *Taxes Which Affect the Consumer*

a. What are "hidden taxes"? Make a study of some.

b. Luxury taxes

c. Sales taxes

ILLUSTRATIVE UNITS

UNIT I

WHAT ARE CREDIT UNIONS AND HOW CAN WE USE THEM?

An Experience Unit

1. *Overview by teacher and planning with pupils*

2. *Objectives developed by teacher-pupil planning*

   a. To understand how resources of a credit union are made available to its members at a reasonable cost and are managed by a democratic process

   b. To find out how credit unions meet the need for cheaper consumer credit for the raised standard of living in the United States

   c. To learn how credit unions represent a cooperative means for promoting thrift and savings

   d. Other objectives developed by pupils and teacher

3. *Learning Activities* (individual, committee, and class work)

   a. What are the purposes and functions of credit unions?

      (1) The promotion of thrift and savings is made possible through small periodic payments.

         (a) Encouragement of thrift

         (b) Reasonable rate of return on savings

         (c) Reasonable rate of interest on loans

         (d) Control of one's own money through democratic cooperation of fellow members

      (2) Accumulation of resources is made available to membership. List various reasons for using the services of a credit union.

   b. Why did the credit union arise and grow in the United States in the field of cooperative financing?

      (1) Cheaper consumer credit in small amounts was needed for consumer goods, new homes, medical expenses, debt obligations. Show how changed standard of living has increased costs of the individual.

(2) How has the credit union movement grown?

(a) Number of credit unions now in operation
Scope and type of membership they cover
Volume of business
Geographical distribution

(b) Make a graph showing development

c. How is a credit union established?
Invite a representative of a nearby credit union to present an outline of the process.

d. How is a credit union set up and operated?

(1) Visit a credit union in a neighboring community. Examine their publications, statements, etc.

(2) Compare the services of the credit union with that of other loan agencies.

(3) Present the ideas and the mathematical aspects in various ways.

4. *Evaluation.* See Chapter IV, page 249.

5. *Suggested Sources of Information*

Giles, Richard, *Credit for Millions.* Harper and Brothers

The Credit Union Bridge (monthly magazine)
Credit Union National Association, Madison, Wisconsin

Federal Credit Union Annual Report (and other free material)
Bureau of Federal Credit Unions
Washington 25, D. C.

Various circulars and literature can be obtained from:
Credit Union National Association, Madison, Wisconsin
or
Pennsylvania Credit Union League, Inc.
4309 N. Front St., Harrisburg, Pa.

UNIT II

FAMILY FINANCIAL SECURITY EDUCATION

A Resource Unit

NOTE: Teachers at other grade levels may find helpful suggestions in connection with problems in living which arise in their classes. For the meaning and use of this unit refer to the section on "Characteristics of a Resource Unit," Chapter II, page 52.

1. *Introduction*

We live in a civilization of constantly increasing complexity which results in unending revision of desires, goals, and standards of achievement. To meet these increasing demands on the individual and on society requires ability to adapt one's knowledge, skills, and attitudes to meet all eventualities. Education is the means by which the individual is encouraged to adjust his attitudes, to develop his capabilities to cope with the problems of living, and to make his contribution to the welfare of mankind.

Competition is a keyword in our democracy of free enterprise, and as the society in which we live develops ever onward to higher levels, so the individual who is to take his place in this competitive, highly developed society must be equipped to an increasing degree with the knowledges, skills, and attitudes which will enable him to compete with others.

To the public school has been delegated the principal role in preparing our children for their places in this complex civilization, and since the school is assuming many duties once left to the home, it is the school that prepares to a large degree vocations and avocations. The school is concerned with the manner in which the student spends his leisure time as well as with the type of employment by which he will earn his livelihood.

A major factor of life adjustment is family financial security education which may well permeate the entire curriculum of any secondary school. Teachers in all subject fields should have an awareness of the needs of the pupils for the knowledge and skills required to obtain an adequate income and to manage the income intelligently by planning a program of spending, saving, and insurance. Since these are universal needs, it is desirable that teachers of related subjects cooperate in utilizing all knowledge, skills, and abilities which contribute to their further fulfillment.

Mathematics is basic to this entire program, and each teacher in carrying out family financial security education becomes, in a sense, a teacher of mathematics. The unique contribution of the mathematics department is teaching the fundamental processes, the use of graphs and tables in expressing and interpreting data, and other skills in number relations in addition to helping the pupil realize the social and economic needs and applications for such skills. But above all there are the understandings of the quantitative and the qualitative relationships in establishing and maintaining one's financial security.

Since the twelfth grade is usually the terminal public school experience for most pupils, it is at this level that the pupil has need for final preparation for the field of employment in which he expects to engage. Employers have in recent years been outspoken in regard to mathematical deficiences in high school graduates. In addition to supplying the arithmetical knowledge and skills required for occupations in the most common types of employment which the graduates will enter, the secondary schools must help their pupils to assume their economic and social responsibilities in the family and in the community as they approach adulthood.

In presenting the various phases of family financial security education, only the very simplest overview can be given. Starting with the pupils' allowances, savings accounts, insurance policies or government bonds, the teacher should stress the importance of planning to meet immediate needs as well as those in the future. Pupils may be encouraged to find out how their own families meet their financial obligations and whether or not they work on a budget. Pupils in high school are old enough to start participating in the family planning, and, if they are allowed to become acquainted with some of the problems which their parents face, they can more readily see their position in relationship to the whole and thus cooperate with their parents as well as receive valuable insight into their own future as adults.

The following plan provides for the study of mathematics as applied to the problems concerning family finance. The initial objectives are highly dependent upon arithmetical knowledge and skills. In the pursuit of this area of study the pupil should also develop a greater appreciation of the principles underlying the "fundamental processes" as well as increase his facility in using them. Varied activities suggest ways in which mathematical skills and methods will enable the pupil to deal more effectively with the financial problems which must be faced by family members.[1]

2. *Purposes of the Unit*

a. Central Objective: Understanding of the means of securing individual and family financial security

b. Contributory Objectives:
   - (1) Knowledge of the sources of income
   - (2) Ability to develop and follow a budget
   - (3) Understanding of the role of savings in family security
   - (4) Knowledge of the ways of borrowing
   - (5) Ability to purchase goods and services wisely
   - (6) Ability to use mathematical skills and knowledge in the solution of problems related to family financial security

c. Indirect Objectives:
   - (1) Ability to interpret information intelligently
   - (2) Habit of estimating and of checking results in situations involving mathematical computations
   - (3) Development of desirable social attitudes, such as respect for the rights of others; appreciation of business integrity; tolerance, fair play, and honesty in all human relations
   - (4) Awareness of the relationships between mathematical competence and occupational opportunity
   - (5) Respect for workers in all types of employment which contribute to the general welfare
   - (6) Appreciation of the contribution of mathematics to the cultural and scientific development of civilization
   - (7) Realization of one's potential value to the community as a person who contributes to the general welfare

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[1] This outline for teachers is adapted from a report which was prepared by a group of mathematics teachers all of whom were members of the Family Financial Security Workshop at the University of Pennsylvania during the summer of 1950.

PROBLEMS	CONTENT
a. Acquiring the family income By what means can the family income be acquired and in what ways can each member contribute to the family security?	a. Acquiring the family income (1) Pay rolls (2) Commission and/or salary (3) Private enterprise or professional services (4) Income from investments (5) Social Security benefits and other pensions (6) Unemployment insurance (7) Take-home pay (a) Income tax (withholding) (b) Social Security or other retirement program (c) Group insurance
b. Distributing the family income wisely How can the practice of budgeting help a family avoid worry and friction over money matters and how can this best be accomplished for your family? (1) How do banks facilitate the payment of financial obligations of the individual and the family? (2) For what purposes does your family spend money and how can your family derive the greatest benefits from these expenditures?	b. Distributing the family income wisely by budgeting (1) Means of paying bills (a) Cash (c) Money order (b) Checks (d) Bank draft (2) Spending the family income (a) Home ownership vs. renting Term insurance Mortgages Fire insurance Taxes Depreciation Maintenance—gas, electricity, heat Interest on investment (b) Economical purchasing Discounts—cash, sales Ratio of quantity to price (c) Credit and installment buying Installment buying vs. cash payment Interest factor in installment buying (d) Insurance as protection Health and accident Hospitalization Automobile Liability Term life

PROBLEMS	CONTENT
c. What are the possible ways of caring for the family savings and what are the advantages and disadvantages of each?	c. Investing the family income and care of securities (1) Life insurance (a) Ordinary life (b) Limited payment life (c) Group insurance (d) Industrial insurance (e) Endowment (2) Annuities (3) Savings accounts (a) Bank savings accounts (b) U. S. Postal Savings (4) Stocks—preferred and common (5) Bonds—government and corporation (6) Real estate mortgages (7) Safety deposit boxes
d. Why is Polonius' advice to his son, "Neither a borrower nor a lender be," not always good advice today?	d. Borrowing to supplement the family income (1) Banks (2) Small loan company (3) Insurance company (on policies) (4) Pawnbrokers (5) Individuals
e. How do the happiness and satisfaction which come to a family through helping others contribute to its own security?	e. Sharing (1) Church (2) Community welfare

4. *Activities*

a Introductory Survey

1. Do you get an allowance? Yes______. No______.
2. Is it between the following?
    $1-$3 per week $3-$5 per week   More______.
3. Are you obliged to spend some of this in any of the following ways?
    School
    Christmas or Purpose Clubs
    Church
    Clothes
    Bank
    Insurance
    College
    Any way you wish
4. Do you have any part-time job? (not summer) Yes______. No______.
5. At this time how much do you earn?
    $1-$5 per week
    $5-$10 per week
    More______.
6. Same as Question 3.
7. Do you have a summer-time job? Yes______. No______.
8. At this time how much do you earn per month?
    Less than $50
    $50-$100
    $100-$200
    More
9. How do you spend it? Clothes Bank College
    Anything else? Name it________________.
10. During a year did you earn between:
    $0-$50 $50-$200 $200-$400   over $400
11. Do you help out your family in any financial way? Yes______. No______.
12. How? Direct money______. Merchandise______. Supporting self______.
13. Do you keep a record of how much you earn; how you spend it during a year?
    Yes______. No______. Tell briefly how.

b Suggested Learning Experience

1. Jimmy said that his father receives three cents for each rivet he puts in a car, and Johnnie said that his father sold a life insurance policy yesterday and received 50% of the first premium paid. Find out in what other ways people earn their income. 4, 32, 33, 51, 56.[1]
2. Since there are other ways of earning money, arrange for visits to the Chamber of Commerce and the Better Business Bureau to gather information on types of employment and rates of pay. 4, 16, 53.
3. A friend said that he earned $350 last month, but he received a check for only $281.20. Study the voucher which accompanies the check and find out what amounts have been deducted and the reason for each deduction. 4, 5, 12, 28, 29, 61. S.[2]
4. The income tax deduction on most salaries is the major item. Sometimes, by the end of the year, these deductions are too large, and sometimes they are too small. From a study of income tax forms, find out why this is true. 69. S.
5. Assume that a family of four has an income of $5400 a year. Make a budget for that family for the year. 3, 4, 5, 7, 10, 18, 27, 37, 42, 49. F, X.

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[1] Numbers refer to items in the Bibliography for Pupil Use, page 177.

[2] Letters refer to items in the List of Films, page 174.

6. In studying the family budget, list the necessary family expenses. What is the simplest way of paying all these bills? (Checks.) Study the essential information on a check and the proper manner of filling it out; then write some sample checks. 3, 4, 7, 10, 24, 18, 25, 38, 49, 62. C, I, O, M, U.

7. Make a budget of one's income and expenditures for a period of not less than two weeks to see if money can be saved by budgeting. Note the sample form on page 8 of the pamphlet, "Budgeting Through School Savings." 3, 4, 7, 10, 24, 18, 25, 38, 49, 62. F, X.

8. When he is in a hurry for athletic goods bought in New York, Mr. Pickett always sends money orders in payment. Why is this wise? From the post office, secure some blank applications for money orders. Find other ways of safely sending money out of the city. C, I, O.

9. Mr. Smith has been paying $95.50 per month rent for a house, and his landlord offered to sell it for $18,000. What expenses other than the purchase price must Mr. Smith assume if he buys the house? What other considerations must he take into account in making his decision? Y.

10. In a conversation overheard the other day on the bus, two women were arguing about tax rates. One said, "Do you know that we're assessed 80%?" The other replied, "No, my husband said our rate is only 2.66%." How could both be correct? P.

11. Do you ever hear your parents complain about high taxes? See how local taxes compare with those in comparable towns or cities. H, P.

12. There is a proverb, "There is nothing certain but death and taxes." Discuss what our community would be like if we paid no taxes. H.

13. What would happen if the local fire department, police, hospitals, etc., were all privately owned?

14. Make a graph giving an idea of the way in which the local tax dollar is spent. 69. P.

15. In the state of Virginia there is a "No Tax Town." How can such a town survive?

16. Bill was saying the other day that his father gave a mortgage on their new home. Explain what was meant. 49.

17. Find a newspaper clipping about the burning of a house. Suppose it had been yours. How would that have affected your family? Study means of insuring your homes against loss by fire. Bring in any expired policies which you may have to illustrate rates and coverage. (This should include fire, tornado, and lightning insurance.) 12, 49. A.

18. Mr. Brown, a customer of a local bank, is getting a mortgage on a home he is buying. Invite a local banker to explain to the class the mortgage form and interest notes. Figure out how to prorate taxes and insurance. 20, 21, 35, 49. C, I, W, T.

19. Now that Mr. Brown has purchased his home, what expenses must he meet in maintaining it? Since he must pay for gas, electricity, and water, visit the utility offices and secure dummy meters and demonstrate how to read them.

20. Keep a record of the readings of home gas, electric, and water meters. Read them again in two weeks and compute the cost. Also examine electric stoves and other appliances to learn the wattage of each and figure the cost of operation.

21. In examining a catalog of athletic supplies, Ted discovered that basketballs were listed at $7.50 less 20% and 10% trade discounts and 2% cash discount. Why does a firm offer two discounts? What would the basketball actually cost Ted? Find out what one discount would be equivalent to 20% and 10%.

22. Search through catalogs to find articles offered at a series of discounts. Find the net price and amount of discount on these articles. Which are the best buys?

23. A 4½ oz. bottle of make-up lotion costs $1.00 while a 10 oz. bottle costs $1.75. Why is it more economical to buy the larger size? Bring some empty cartons and labels to class to study the ratio of quantity to price and determine the better buy by using proportion. Under what conditions it it not advisable to buy in large quantities? 4, 14, 15, 51. K, V.

24. Why are chain stores able to sell at lower prices than your local merchant?

25. What are some of the hidden costs that have to be passed on to the customer? 51.

26. Collect and analyze proverbs concerning economy. Example: "A penny saved is a penny earned."

27. The information from the tag on a refrigerator reads: "$185 cash or $50 down and $15 a month for 10 months." Figure the extra cost of buying this on the installment plan and the yearly rate of interest. 42, 51, 78; D, Y.

28. Visit local stores and bring back information regarding cash price and installation price for buying certain articles. 51, 78.

29. An advertisement about articles for sale states: "10% down and $5.00 a week." Investigate how merchants can stand the extra costs involved in the delayed collection of such accounts. Is it really as cheap as paying cash to buy this way?

30. Find out if a merchant will sell on installments at the same price he will sell for cash. Would he be losing money if he did? *Note:* In spite of the fact that a merchant will often advertise, "No charge for credit," he will probably reduce the price if offered cash.

31. Most people buy automobiles on the installment plan. What are all the expenses involved in such a purchase? 4. K.

32. Organize a panel discussion on the advantages and disadvantages of installment buying. 49. D, Y.

33. Since in many states automobiles must be insured, let us study all types of insurance which would or could be carried on autos. 12, 72.

34. Select a committee to make a report on automobile insurance rates. The committee might select about six typical cars, make a record of the make, model, and year, and then take the list to an agent for an automobile insurance company to find out what premium rates the company would charge for the five different types of insurance:—fire, theft, collision, property damage, and personal liability.

35. Make a chart for the bulletin board from these rates. Figure the total cost of insuring the six cars. 72.

36. It is said that "insurance spreads risks." Show how this is accomplished by studying the history of insurance. On what types of insurance is this true? 3. Q, R.

37. Some schools offer group accident insurance. What is the coverage of such a policy? Find out if there are any other types of health and accident insurance offered in the local school system. 49.

38. In such policies there is a definite premium for this insurance coverage during a limited time, such as one to three years. How does life insurance compare with the other types of policies? 71.

39. Investigate how term insurance gives financial protection to the family in case of death during the time insured. What other types of insurance will contribute to family security? 3, 23, 64, 67, 73. B, J.

40. Make a comparison of the provisions of the different types of insurance policies. 3, 7, 23, 30, 32, 43, 49, 64. B, J, Q, R.

41. Invite a life underwriter to visit the class to tell more about insurance and to answer any questions which may arise. 7, 32, 44, 45, 51, 52, 64, 67, 73.

42. Have each pupil print on paper to be handed in the estimated amount of the family income, number of children in the family, age of each child, and whether or not the father pays for social security. This paper is to be unsigned and folded twice before it is handed in. Have a committee of five examine these and select three different situations so that the visiting life underwriter can explain the insurance plans he would suggest for these families. 5, 7, 12, 29, 32, 33, 44, 45, 51, 61, 64, 67, 73, 85. B.

43. Study the pamphlet *A Date with Your Future,* to see what policies are suggested to meet the needs described there. 3, 7, 12, 23, 30, 32, 36, 37, 44, 51.

44. It has been said, "Insurance is certain small installments on an uncertain large cost." Apply this statement to the types of insurance studied in class to see if it is true. 3, 7, 23, 30, 32, 44, 49, 51, 52, 67.

45. Develop an appropriate skit or use the play *But Definitely* as an assembly program. 7, 12, 5, 11, 23, 30, 32, 44, 51, 52.

46. Investigate other forms of savings and compare these with life insurance as t safety protection, interest, and other advantages. 3, 5, 7, 21, 23, 24, 32, 37, 38, 44 49, 52, 51, 57, 62, 67, 73, 76. C, T.

47. Mr. James says that he deposits money in his savings account until he gets abou $1000 and then buys bonds or shares of stock. Study these other forms of investments. 7, 21, 24, 32, 37, 38, 51, 52, 54, 57, 73. N.

48. Study the steps necessary in establishing a new business and the means of interesting other people in investing in its stock. 22, 25, 37, 49, 74, 86.

49. View the filmstrip "The Stock Exchange." 1, 22, 24, 37, 40, 55, 86. a.

50. Bring in some copies of newspapers for the past week. Find the Stock Marke Quotations. Why is a stock market important in our economic system?

51. Assume that each member of the class has $1000 to invest. Plan to invest in a least three different types of stock. Follow the quotations in the daily paper an keep records of the fluctuations for six weeks. See who made money either o dividends or on a rise in price. Show the wisdom in studying a business befor investing in it. 1, 22, 24, 26, 40, 55, 86.

52. Arrange a visit to one of the city brokerage houses to see how stocks are bough and sold. 1, 22, 24, 26, 40, 55, 86.

53. Show the filmstrip "The Stock Exchange" in order to observe a complete stoc transaction. 1, 55. a.

54. Assume that an industry which members of the class organized has been operatin for two years and the net earnings have been so high as to make possible th paying of fairly high dividends. What would be the effect on the market pri of this stock? Since the product is selling so well, would it be wise to enlarge i facilities? Study the means by which additional funds can be secured. H would the situation be changed if "the class industry" had been losing money f two years? 1, 22, 17, 40, 49, 86.

55. Choose a committee of three to investigate the different types of corporation bon in which people invest. Study these bonds from the standpoint of relative saf and rate of return. 1, 22, 17, 25, 49, 86.

56. Have a committee report on the manner in which our Government secured fun to finance World Wars I and II; compare and contrast the types of bonds issu and the rates of return. 1, 10, 17, 24, 49, 51, 70.

57. Investigate other kinds of government bonds (municipal, state, and nation from the standpoint of relative safety of the investment, rate of return, and t types of people who invest in them. 1, 10, 17, 24, 49, 51, 70, 76.

58. Under what type of contract could a man borrow money if he wanted to buy home and didn't have sufficient funds? (Mortgages.) 17, 19, 24, 35, 42, 49.

59. Organize a panel discussion on the comparative safety and rates of return of stocks, bonds, mortgages, and life insurance. 17, 35, 49.
60. Report on instances where a person suffered loss because he was careless with valuable papers. What means are available for protecting these papers? (Safety deposit vaults.) What are the special precautions taken by those who operate the safety deposit vaults to protect the valuables which are placed therein? 17, 35, 49, 58. C.
61. Discuss occasions when individuals might wish to borrow. Become familiar with sources from which one can borrow and the rates of interest that would have to be paid. 17, 35, 49, 58, 78. E, I, T.
62. List the sources from which one might borrow. (Bank, small loan company, insurance company, pawnbroker, or individuals.) Divide the class into committees to study the manner in which these agencies operate and the rates charged by each. 17, 20, 35, 58, 78. E, I, W, T, C.
63. List and discuss the ways in which families share their time, possessions, money, and talents.

5. *Types of Evaluation*

a. Improved Family Understanding (mimeographed form to be sent home near the end of the term)

Family Finance Evaluation

During the past term your son or daughter has been studying problems in Family Finance. The school would appreciate your cooperation in filling out this form so that we may know how practical the course is.

1. Does the student manifest greater interest in family finances?
   Yes______. No______.
2. Does the student participate in planning family expenditures?
   Yes______. No______.
3. Does the student manage his funds wisely by:
   a. Showing improved judgment in selecting goods?
   Yes______. No______.
   b. Taking better care of articles purchased?
   Yes______. No______.
   c. Keeping within his allowance and/or earnings?
   Yes______. No______.
   d. Saving a portion of his allowance and/or earnings?
   Yes______. No______.
   e. Sharing his allowance and/or earnings with others?
   Yes______. No______.
4. Do you feel that this course has been of value to your son or daughter?
   Yes______. No______.
5. What suggestions can you make as to the content of this course which would have made it more valuable to your son or daughter?

b. Sample Test Items:

I

True or False Test

Place T or F before each statement to indicate whether it is true or false

1. One should take out life insurance when he is young and in good health.
2. The law of averages is based on the study of a very large number of cases.

3. A term policy pays proceeds in the form of life income.
4. Ordinary life insurance can be used not only for protection but also as a means of accumulating money for use in later life.
5. A term policy may be renewed at the end of the term at the same rate.
6. The amount of premium paid varies with the type of policy and the age of the insured.
7. Group insurance is cheaper than ordinary insurance.
8. Life insurance companies make money by speculating in stocks.
9. Insurance is the transfer of small certain loss for large uncertain but possible loss by payment of a premium.
10. When you buy a share of stock you become one of the owners of the corporation.
11. When the Federal Government wishes to borrow money it sells shares of stock
12. If a share of preferred stock is bought above par, the rate of income is highe than the rate of dividend.
13. In general, the higher the rate of interest or dividend on an investment, th safer the investment.
14. If a house has a mortgage on it, the owner has borrowed money and pledge the house as security.
15. Government bonds are considered one of the safest investments at the presen time.
16. Before buying a mortgage a check should be made to see that there is n indebtedness on the property and that the taxes have been paid to date.
17. A 6% preferred stock pays 6% of its face value.
18. A 6% bond pays 6% of its face value.
19. The par value of a share of stock does not change.
20. A savings account in which regular deposits are made is a good investment f young people.

II

Matching Items

Match statements at the left with correct word in the right column:

1. The type of insurance with the largest savings feature.
2. The agreement between the insurance company and the person taking out the insurance.
3. The amount paid for insurance.
4. The type of insurance with no saving feature.
5. The person who sells life insurance.
6. The amount some insurance companies return to their policy holders if they collect a larger premium than needed.
7. The person who receives the money from an ordinary life policy when the insured dies.
8. What you would buy if you did not want protection but wanted income for your old age.
9. The type of insurance taken out with fellow employes.
10. The type of insurance which is usually paid for each week when the collector calls at your home.

a. Policy
b. Term
c. Annuity
d. Dividend
e. Life underwriter
f. Endowment
g. group
h. Industrial
i. Beneficiary
j. Premium

III

Completion

1. A married man with three children and an average income would probably take out __________________________ insurance first.
2. An unmarried working woman with no dependents would probably take __________________ type of insurance.
3. A married man with two children and above average income would probably want ______________________ policy.
4. A married man with five children and low income would probably take ____________________________ insurance.
5. If premiums on a life insurance policy have not been paid within the required time, the policy is said to have ______________________.
6. Your parents would probably take out ____________________ policy if they wished to save for your college education.
7. The price paid for a share of stock is called its ______________ value.

IV

Problems

1. Find the total cost of 25 shares of J. I. Case stock selling on the market at \$125¼. Brokerage charge is ⅛% of the total market value.
2. What would be your total income from three mortgages—one for \$2,500 at 5½%, one for \$4,000 at 5%, and one for \$3,600 at 6%?
3. Find the rate of income on 6% stock, par value \$100, which was bought for \$90.
4. How much annual interest would you receive from a \$1,000–4½% bond?
5. List four things you would find out about a corporation before you would buy its stock.
6. The following problems illustrate the application of various arithmetical skills to facts in the C.S.O. Mortality Table.[1]
   a. The mortality table shows that of 1,000,000 babies one year of age, only 157,799 will live to be 81 years old. How many will die before reaching that age?
   b. 837,413 lived to be 47 years old but only 454,540 became 70 years old. How many died between these two ages?
   c. 9,990 persons died during their 50th year, 10,628 in the 51st year, 11,301 in 52nd year, 12,020 in 53rd year, and 12,770 in 54th year. How many persons died during the 5 years? What was the average number dying each year?
   d. Of the 1,000,000 babies aged 1 year, 958,098 lived to be 17 years old. What percentage of the babies lived to that age? Express as a common fraction reduced to lowest terms and as a decimal fraction to the nearest thousandth.
   e. Although 960,201 of the babies lived to be 16 years old, 2,103 died during that year. How many lived to be 17 years old? What percentage of those 16 years old died? (Express to the nearest tenth of one per cent.)
   f. Of the 829,114 who lived to be 48 years old, .01064 died during that year. How many persons died while 48 years of age? During what year of their lives did they die?
   g. If 125 lived to be 99 years old and 125 died during that year, what per cent of those living at the beginning of the year died during the year?

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[1] Commissioners Standard Ordinary Mortality Table.

h. It is noted that .99775 of those living at the beginning of age 17 survived to age 18. This is how many per 1,000? This is what per cent to the nearest tenth?

i. Of the 620,782 who lived to be 63 years old, 20,958 died. What per cent of them lived to be 64?

j. If 92.45% of those who lived to be 73 died during that year, what per cent lived through the year?

k. 740,631 persons lived to the 57th year, but 14,390 persons died during that year, 15,251 died during the 58th year, 16,147 died during the 59th year, and 17,072 died during the 60th year. What was the probability of dying within the four years? What was the probability of living throughout the four years?

7. The annual premium on a 20-year endowment policy at age 25 is $40.51. What would I pay annually on a $5,000 policy? On a $6,500 policy?
8. If a man received a $14.97 dividend on his annual premium of $99.80, what per cent of the premium was the dividend? What was the net cost of the policy that year?
9. The annual premium on a 15-payment limited pay life is $102.36. If the rate is $25.59 per $1,000, what is the face of the policy? What will be the total cost of the policy for the 15 years?
10. At age 20 I can get a 10-year term policy at $9.70 per $1,000. What wil $7,000 of this insurance cost me for the ten years? How much will I receive from the company when the policy matures?
11. The annual premium on whole life policy at age 25 is $16.68. The semi-annua premium is .52 of the annual premium. What will be the extra yearly cos if I pay semi-annually instead of annually?
12. The annual premium on a 20-year endowment policy at age 25 is $42.41 pe $1,000 but at age 26 it is $42.50 per $1,000. At the end of the 20 years ho much would be saved by buying the policy at the younger age?
13. If you pay $.10 weekly on a $240 industrial policy taken out at age 10, whic matures at age 65, what will be the total cost? How much will you receiv when the policy matures? What will be the cost of protection during the 5 years?

6. *Source Materials*

a. FILMS AND FILMSTRIPS

Films*

A^1 *A Word to the Wise.* 15 min.
National Retail Mutual Insurance Co., Chicago.
This film tells about fire causes and prevention. It tells a little about fi insurance.

B^1 *American Portrait.* 25 min.
Free loan—branch offices, Association Films. Purchase—Institute of Life Insu ance, 488 Madison Ave., N. Y. 22.
Starring Alan Ladd, this film portrays the role of salesmanship in introduci a better way of life and what life insurance contributed to family life duri the last century.

C^3 *Banks and Credit.* 10 min.
Purchase—Coronet Films, Coronet Bldg., Chicago 1.
Brings out the essential part a commercial bank plays in the economic life of community. It explains the important part credit plays in our econo system.

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* ^{3} Excellent, ^{2} Good, ^{1} Fair. These are the ratings of the original workshop committee on Fam Financial Security.

D² *Distributing America's Goods.* 11 min.
Purchase or rent—Encyclopaedia Britannica Films, Inc. Wilmette, Ill.
Comparison of values, cost of distributing and advertising of goods. Shows methods of economical buying.

E² *Every Seventh Family.* 26 min.
Free loan—Modern Talking Picture Service, 9 Rockefeller Plaza, N. Y. 22.
Story of consumer finance service, the needs which created it, and the important role it plays in the community.

F³ *Family Life.* 10 min.
Purchase—Coronet Films, Chicago.
Creates an awareness of the happiness to be gained from a well-managed home, and outlines a program for achieving it.

G³ *The Federal Reserve Bank and You.* 30 min.
Free loan—Federal Reserve Bank.
How the Federal Reserve System helps business, affects personal life, aids member banks, and how it functions.

H¹ *Federal Taxation.* 10 min.
Purchase—Coronet Films, Chicago.
Film tells how the country gets and spends its tax dollar.
It also tells what we get from taxes.

I¹ *Fred Meets a Bank.* 10 min.
Purchase—Coronet Films, Chicago.
Inside story on banking procedures. Fred learns about banking, checks, opening savings accounts, and securing loans. Tour behind the scenes through the different departments of a bank.

J² *For Some Must Watch.* 28 min.
Free loan—Castle Films. Purchase—Institute of Life Insurance.
488 Madison Ave., N. Y. 22.
True documentary film showing the sympathetic understanding, wisdom, and help of a life insurance underwriter in a typical American community.

K¹ *Getting Your Money's Worth.* 10 min.
Free loan—General Motors Free Library, Detroit.
Tells what to test for in purchasing a secondhand car.

L¹ *How Banks Serve.* 11 min.
Local chapter of American Bankers Association.
Presents the three basic functions of commercial banks; safeguarding money; transferring funds; making loans.

M³ *Making Money* and *Know Your Money.* 20 min.
Rent—Business Education Visual Aids.
First film tells of processes of making money in the U. S. Second film tells of steps to take in detecting counterfeit money.

N² *Money at Work.* 17 min.
Free loan—Modern Talking Picture Service, 9 Rockefeller Plaza, N. Y. 20.
Presents a panorama of American industry. Shows why the Stock Exchange exists, how it plays its part in democratic economy. New York Stock Exchange.

O¹ *Pay to the Order of.* 12 min.
Local Chapter of American Bankers Association.
Shows the four main reasons for using checks, how to write and endorse checks, and explains the journey of a check in completing a payment.

P¹ *Property Taxation.* 10 min.
Encyclopaedia Britannica Films, Inc., Wilmette, Ill.
Gives the arithmetic of taxes. Tells difference between true value and assessed value.

Q^2 *Search for Security.* 17 min.
Free loan—Association Films or Castle Films. Purchase—Institute of Life Insurance, 488 Madison Ave., N. Y. 22.
Traces the development of life insurance, explains the operation of modern life insurance, and how it meets the needs of individual and family.

R^2 *Sharing Economic Risks.* 10 min.
Coronet Films, Chicago 1, Free loan—Association Films.
Explains the nature of economic risk, and describes many of the risks we face in our daily lives. Shows how families use insurance to protect themselves from loss resulting from risks. Defines insurance terms.

S^1 *Social Security.* 10 min.
Teaching Films Custodians, Inc., 25 W. 43rd St., N. Y. 18.
Interprets the operation of the Social Security Act from the time of its passage through its various steps to the final payment of checks to the beneficiaries

T^2 *Using a Bank.* 10 min.
Encyclopaedia Britannica Films, Inc., Wilmette, Ill.
The story of a man starting a new business. Presents the various services o a commercial bank, giving special stress to savings, checking, and loans.

U^1 *What is Money?* 10 min.
Rent—Swank Motion Pictures, Inc., 614 N. Skinker Blvd., St. Louis, Purchase—Coronet Films, Coronet Bldg., Chicago, 1.
Shows how money functions as a standard of value, standard for future pay
ment, storehouse of value, and convenient medium of exchange for goods an
services. Evolution of monetary systems from days of primitive barter, an
how checks work today as a substitute for money.

V^2 *Wise Buying.* 11 min.
Purchase or rent—Coronet Films, Coronet Bldg., Chicago 1.
Shows how a young girl learned to buy clothes that would last and still kee
within her budget.

W^1 *Work of the Stock Exchange.* 16 min.
Purchase—Coronet Films, Coronet Bldg., Chicago 1.
Each step of incorporation and listing of stock is given. Details of buying a
selling operations on the exchange floor and in the broker's office. Shows h
these operations bring to land, labor, and management the capital necessa
for production.

X^1 *Your Thrift Habits.* 10 min.
Purchase—Coronet Films, Coronet Bldg., Chicago 1.
Develops a clear definition of thrift. Presents the need for a budget a
emphasizes the importance of regular savings, and the making of decisions.

Y^2 *Installment Buying.* 10 min.
Purchase—Coronet Films, Coronet Bldg., Chicago 1.
Demonstrates some of the pitfalls of installment buying, and encourages a co
plete investigation of installment credit, contracts, and interest rates.

Filmstrips

a^1 *The Stock Exchange*
New York Stock Exchange, 11 Wall Street, New York, N. Y.
Part I shows the nature and function of the Stock Exchange, describes its
ganization and its economic role. Part II shows how it operates, and follow
typical stock sale between two individuals living in different parts of
country.

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UNIT III

HOW SHALL WE TAKE CARE OF OUR MONEY?

1. *Overview by Teacher and Planning with Pupils*

a. Teacher's Presentation (suggested)

"Nearly every one of us has used the services of a bank. Money problems are experienced by us at five and six years of age and continue throughout life. It is the purpose of this unit to familiarize us with banks and to provide us with the mathematical tools to cope with simple financial problems and to make us intelligent consumers.

"Some of us are interested in money problems, having acquired a pride in saving when quite young. We will gain further experience, driven by our interest. This type of unit will provide us with the necessary tools to broaden our interest.

"It is necessary for all of us who reach an age where we can begin earning and spending our own money to be made aware of certain fundamentals that will protect us.

"The cause of most crimes and poverty is ignorance. If we are to be prepared for life we must be prepared to think clearly in money matters. The probable necessity, immediately upon leaving the high school, of (1) borrowing money, (2) financing a car, or (3) starting a checking account is fundamental and taken for granted. Such situations will arise for the majority of us. We need preparation for this now.

"This unit on banking can be learned from many viewpoints. The best point of view may well be that of the consumer. The unit is divided into four phases: (1) savings accounts, (2) checking accounts, (3) borrowing money, and (4) bankers' finances. It is this last phase where we should round out the picture by getting a glimpse of what goes on behind the counter and how it affects our status as consumers.

"In all of these phases there will be opportunity for us to use such forms as deposit slips, withdrawal slips, checks, promissory notes, etc. We should learn about little legal technicalities, especially in filling out checks.

"Let us see now what we know and what we can learn from this unit."

b. Pretest—To create a sense of problems

1. Fill out the following check as if you were paying for a $49.95 bicycle that you bought from me.

Town, Pa. ____________________19____ No.______

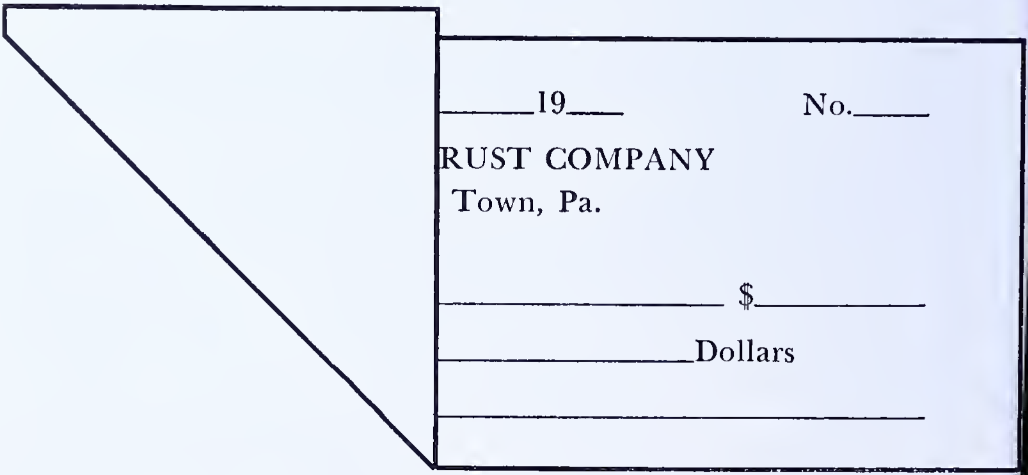
GOOD TRUST COMPANY
of Town, Pa.

Pay to the
order of ________________________________ $____________

____Dollars

____

2. How much money have you saved to date? ____________________
3. What is the total amount of your assets? ____________________
4. If you put $50.00 in the bank at 2% simple interest 6 years ago, would you have a sum large enough in 1952 to buy an $80.00 car? ____________________
5. If you had to borrow some money, where would you go to get it? ____________________



________19____ No.______

RUST COMPANY
Town, Pa.

____________________ $____________

____________________Dollars

____

6. Here is a check. If you were Johnnie Jones how would you endorse it? ____________________
7. How would you endorse it? ____________________
8. If you borrowed $10.00 from a friend and he asked for $.10 a week for the use of it, how much interest would you be paying? ____________________

9. How does that compare with a bank that loans money at 5%? ______________________
10. What is a "silver certificate" commonly called? ______________________
11. Do you know what the bank does with the money the people deposit? ______________________
12. If in 1940 you put $3.00 in the bank at 1½% compounded semiannually, how much could you withdraw in 1950? ______________________
13. What rate of interest does a buyer get in U. S. Bonds? ______________________
14. Do you know why we can get interest on our money when we put it in a bank? ______________________
15. Do you know the difference between *credit* and *money?* ______________________
16. What is the approximate prevailing rate of interest on savings? ______________________
17. Is there anyone in the class who has a father or near relative in one of the local banks?
18. A woman borrowed $50.00 from a loan shark and agreed to pay $10.00 a week interest and ended by paying a total of $4,395.00, with $400 yet due in a little over a year's time.
19. What banks have we in ______________________
20. Why are banks necessary?
21. Story of John D. Rockefeller's discovery when a boy that by saving $50.00 he could get as much in interest by having his money work for him as he could by several days of hard work.

c. Other Initiatory Activities

1. Have a short discussion or autobiographic essay on personal experiences with banks.
2. The film *Money to Loan* would be a good initiatory activity. It dramatizes how swindlers can take advantage of careless consumers, but gives no details concerning how to avoid being swindled.
3. The film *Story of Money* might be used.

2. *Objectives* (produced by teacher-pupil planning)

a. Central Objective

To develop the ability to use the services of a bank

b. Contributory Objectives

(1) Understanding the bank's role in the community
(2) Understanding the services offered to the public
(3) Ability to solve simple financial problems involving:
    (a) Simple interest and bank discount
    (b) Compound interest
    (c) Nominal and effective rates of interest
    (d) Determination of unknown time and unknown rate

(4) To understand the technical language and arithmetical skills involved in financial problems

(5) To understand the fundamental principle of interest expressed in formulas and applied to practical situations

(6) To gain increased confidence in planning financial undertakings and in using technical terms

(7) To acquire appreciation of the fluidity of money and of how the bank acts as a clearing house for business

(8) To acquire an intelligent consumer attitude about financial matters (Example: It's better to *save* for ______ than borrow to pay for ______)

(9) To increase knowledge of the occupations in bank work

(10) To increase self-confidence in money matters

(11) To increase awareness of "shady" schemes that take money from people

(12) Other objectives suggested by pupils

3. *Learning Activities* (Individual, committee, or class work with reports)

a. A committee may arrange with one of the local banks for the class to visit the bank. The group could plan with a bank officer an efficient and impressive tour, with remarks by the officer that would emphasize the need for familiarization with financial matters. (This might be used as an initiatory activity. Some preparation on the pupils' part would be valuable.) Follow-up discussion will lead to step b.

b. Establish a class bank in which to carry on financial transactions. Organize a company, vote capital stock, and elect officers. Have a teller's desk in one corner of the room. Have a bookkeeping file, check books, deposit slips, etc. The project can be carried out without using actual money.

c. Using a month as a conversion period will allow for the figuring of interest on savings and loans. Have a cashier and bookkeeper in charge; have deposits received and checks cashed. Continue operations until each child has had the opportunity of having two days' banking experience—one as cashier, and one as bookkeeper. The two new clerks could be coached by the two clerks going off duty, who could act as assistants. The bookkeeper should balance the day's accounts.

At the end of this time (approximately one month), have each depositor make his own bank statement, showing deposits and withdrawals; and draw a final check to balance the account.

This activity might take two or three full periods to organize and then be shortened to a 10-minute routine at the start of the class. This activity then would be the source of new types of problems for the children that could be studied during the remainder of the period. It is up to the teacher to see to it that all common types of financial problems are brought up for study.

d. Three basic types of financial situations should arise. There need be no set sequence of these types but they will probably arise naturally in the following order:

(1) SAVINGS

(a) References (See page 191.)

Applied Business Arithmetic, Unit 1
Making Sure of Arithmetic, simple problems
Using Dollars and Sense, pp. 204-208, 227, 230, 231, 280-282
Elementary Mathematics of Finance, Chapter 1
Teaching Thrift in Terms of the Future, article
Mathematics of Everyday Life, Finance Unit, pp. 21-24, 82, 90.

(b) Film:

Fred Meets a Bank (See page 192.)

(c) Material:

Bank forms, passbooks, deposit slips, etc.

(d) Notes for teacher

Here questions should be asked about interest in terms of how much will I have by such-and-such a date, or how long will it take my money to increase enough to buy so-and-so? It might be good to have the pupils make a personal book of original problems in which they are interested. (This would continue throughout the unit.)

It should be brought out that the decimal point, although not so named, is used in writing money numbers which express fractions of a dollar; that the decimal principle of our number system can be extended to express certain parts of a unit which have formerly been expressed by common fractions whose denominators were powers of 10. Immediately the children should begin to understand, to read and to write decimals of one, two, three, or more places separately and in conjunction with whole numbers.

(e) Sample problems for saving

The sample problems will be used to broaden the scope of the pupil's thinking and give him practice. Most of the pupil's problems should be his own.

(1) John deposited $3.56 in the class bank. He wants to know what the total sum should be at the end of the month if the bank pays 5% interest per month.

Answer: $3.58; that is $3.56 plus .0178

(2) If a 25-year-old man can save $50 a month and deposit it in a savings account paying 2% interest compounded semiannually, how much will he have when he is 65 years old?
Answer: $58,713

(3) The Good Trust Co. will pay 1% per annum and the Savings Bank will pay ½% semiannually. Which gives the higher rate of interest?
Answer: Savings Bank

(2) CHECKING ACCOUNTS

(a) References: See page 191
Mathematics of Everyday Life, Finance Unit, pp. 82-90
Applied Business Arithmetic, Unit V
Work Book in Business Arithmetic

(b) Film:
Check and Double Check

(c) Material:

Checkbooks, bank statements, promissory notes of various types, etc.

(d) Notes for teacher

Under this phase will arise the bookkeeping type of arithmetic. Here the pupils can be shown the need for rapid calculation. There are a few short cuts and tricks in ordinary addition and subtraction involved in bookkeeping that would interest the pupils. Time must be spent in rapid drill to emphasize the short cuts of this new method in their subsequent operations so the skill is not lost.

It would be worth while to discuss checking accounts pro and con, and also to compare the services of various banks.

The forms and routines involved in the use of checks should be dramatized and made clear. Teach the pupils why checks, including the stubs, should be filled out carefully. It should be the responsibility of the cashier of the class bank to watch for poorly filled out forms in all services.

(3) BORROWING MONEY

(a) References:

One Hundred Problems in Consumer Credit
Using Dollars and Sense, pp. 182, 190-198
Mathematics of Everyday Life, pp. 92-97
To Tell You the Truth
Facts You Should Know About Borrowing
Short Term Credit a Good Farm Tool
Using Credit Instruments

(b) Film:

Money to Loan (Crime Does Not Pay Series)

(c) Materials:

Forms, monthly installment-charge charts, brochures from finance companies

(d) Notes for teacher:

This phase is likely to be the most difficult and therefore the longest part of the unit. It is a phase of financial experience with which the pupils are not too well acquainted. They probably have done much borrowing, and it is possible to capitalize upon this experience. Haven't the pupils at times had to do special favors for the use of some article or money which they wanted?

The teacher could set up the rate of interest on loans in the class bank and have members of the class go to various places that loan money and bring in material to compare rates.

4. *Evaluation*

a. When the three services of the bank are covered there should be a performance test of achievement. The teacher could observe as the children run through the various routines. While they wait their turn they could be working on some problems the teacher provides which cover the three phases. Reviews may be necessary.

b. Bankers' Finances

When the pupils have each served as cashier and bookkeeper, the class bank could be studied from the banker's point of view. This could be in preparation for the dissolution of the bank. Here the pupils must see some of the problems a banker faces and the type of arithmetical operations involved. In operational performance the child should by now see how to determine the reasonableness of an answer with reference to the decimal point and begin to feel confident in doing any basic operations involving decimals.

Example: Suppose a manufacturer of radio cabinets comes to the bank to borrow $10,000 for some special wood. He shows the bank an order he has received for 1,000 radio cabinets at $20 apiece. The manufacturer says the production costs will be about $5,000. Suppose the bank takes his note, that is, his promise to pay back the bank within 3 months and credits his account with the $10,000, interest to be at the rate of 6%. Would the manufacturer be able to make his desired profit of 15% on his $10,000? Is the manufacturer a safe risk from a banker's point of view?

Answer: Manufacturer makes 20.48%

The film, *Using the Bank,* page 192, could be used here to introduce the follow-up discussion and final evaluation.

c. For the final evaluation the members of the class could prepare two formal statements, one of the bank's condition while in operation, and one of their own financial condition. Here the children could study various financial records to familiarize themselves with formal financial statements and to learn how to interpret them. These final reports must be accurate and presentable for they are to be taken home to the pupils' parents.

d. As a supplemental evaluation, the teacher could provide the children with some sample problems covering the unit's work.

e. Record ways in which bank services are utilized.

f. Other means of evaluation suggested by pupils.

g. Formal drill and remedial teaching where evaluation shows a need.

5. *References*

a. *For Pupils*

American Education Press, *The Banks and You.* Unit Study Booklet No. 13, The Educational Printing House, Inc., Columbus, Ohio, 1933.

American Enterprise Association, *Who Gets the Money?* 400 W. Madison Ave., Chicago, Illinois.

Boyce, G. A., and Beatty, W. W., *Mathematics for Everyday Life.* Finance Unit, Inor Publishing Company, New York, 1936.

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Jansen, Antoville, and Trube, *Practical Mathematics.* Noble and Noble Publishers, Inc., 100 Fifth Ave., New York, 1942.

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Thomson, Margaret M., *Money and Banks.* Basic Social Education Series, Row, Peterson and Company, New York. 1942.
Good explanation of bank work.

Washington Information Bureau, *Banking System in U. S.* Washington, D. C. Free.

b. *For Teachers*

American Bankers Association, *Changes in Bank Earning Assets.* Economic Policy Commission, 1939 (Statistics for "Bankers Finances")

American Bankers Association, *The Bank Chartering History and Policies of the U. S.* Economic Policy Commission, 1939.

Beale, Frank S., Lehigh University, *Elementary Mathematics of Finance.* Edwards Brothers, Inc., Ann Arbor, Michigan, 1947.

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Clark, Harold F., "Teaching Thrift in Terms of the Future," *Child Study Magazine,* New York, May, 1932.

Crook, William F., *Do You Want to Become a Banker?* Frederick A. Stokes Co. New York, 1939.

Hart, William L., *Tables for Mathematics of Investment.* Third Edition. D. C. Heath and Company, Boston, Mass.

Smith, Joseph W., *Workbook in Business Arithmetic.* South-Western Publishing Company, New York. Drill and tests in rapid calculation.

Making Sure of Arithmetic. Silver Burdett Company, New York. Simple Problems.

6. *Films*

BANKS

Fred Meets a Bank—Coronet, 1947. 10 min. sd. $45.00

Using the Bank—EBF, 1947. 10 min. sd. (excellent)
Arith. series. $45.00, rent $2.50

CREDIT

Money to Loan—TFC 22 min. sd. (Crime Does Not Pay Series)

MONEY

Story of Money—Int. Film Bur., 1946. 16 min. sd. $47.50, rent $2.50. British

FORGERY

Check and Double Check—Aetna, 1945. 17 min. sd. color *loan*
Specific suggestions for check losses.

NOTES

SPECIALIZED MATHEMATICS

ALGEBRA I

GRADE NINE

THEME: *Mathematics for the Individual*

ATTENTION TO BASIC MATHEMATICS

There is general agreement that first-year algebra, or the introductory algebra course, should begin in the ninth grade, although the pupil will have some familiarity with problems, equations, and formulas in grades 7 and 8.

Algebra, to the pupil who first begins its formal study, is an extension of a familiar "language." Therefore, the transition to the new concepts and skills should be made as extensions of similar concepts and skills used in arithmetic.

This course of study is not built around a particular textbook but is prepared so that any good text or texts which include the stated topics may be selected for pupil and teacher use in making this course of study effective.

The wide range of individual differences in every class should constantly challenge the teacher to understand each child as an individual. Materials of instruction should be so selected and presented as to make possible adaptation of them to existing individual abilities and interests. (See section on Individualizing Instruction, Chapter II, page 59.)

Pupils should know that first-year algebra is usually a requirement for college entrance or entrance to other institutions of higher learning and is a universal language for the mathematician and the scientist. It is impossible to study higher mathematics without algebra as a foundation.

Pupils who are ambitious to become well-informed men and women, will want to know something beyond elementary arithmetic about a subject that has helped to advance civilization. Now is the time to help these pupils plan a broad program for their education so that they may live useful and interesting lives.

Whenever feasible, problems involving the various algebraic concepts being learned should include problems dealing with mathematics for the individual as outlined in Basic Mathematics I.

Basic Mathematics. It should be emphasized that the essence of this guide is for *all* pupils to have training in the common learnings of mathematics. In a specialized course, such as algebra, the teacher should give attention to the mathematics for daily living by:

(1) Studying and utilizing the outline for the Basic Mathematics suggested for this same grade level

(2) Emphasizing practical applications of the topics in algebra

(3) Stressing particularly the understandings in those topics marked "Basic" in the Outline of Activities by Topics below. Such topics have comparable areas in Basic Mathematics. The essential difference is that in algebra the concepts and techniques will be extended further than will be done in the similar topic as presented in the Common Learnings Program.

OBJECTIVES OF ALGEBRA I

The following objectives of first-year algebra are the goals for pupil development, and therefore are guides to teachers in helping pupils to:

1. Broaden and strengthen the fundamental concepts developed in arithmetic and gradually make the transition to a workable understanding of the terminology, notation, and symbolism of algebra
2. Orient pupils' thinking and their knowledge to the mathematical needs of a good citizen and intelligent consumer
3. Develop logical thinking in developing a problem-solving approach in connection with everyday situations
4. Learn to formulate and evaluate formulas
5. Translate with understanding data into an equation and solve the equation
6. Understand the fundamental concepts of algebra and use with accuracy and reasonable speed the processes involved
7. Estimate with reasonable exactness the desired answer and check thereby the results of all computations
8. Appreciate the function of algebra in everyday life, in the sciences and as a foundation for higher mathematics

OUTLINE OF ACTIVITIES BY TOPICS

TOPIC I. Orientation—Basic

TOPIC II. Formulas—Basic

TOPIC III. Directed Numbers and Fundamental Operations—Basi
(Exclude polynomials as divisors.)

TOPIC IV. Graphs—Basic

TOPIC V. Linear Equations in One Unknown—Basic

TOPIC VI.	Linear Equations in Two Unknowns
TOPIC VII.	Special Products and Factoring
TOPIC VIII.	Operations with Fractions—Basic
TOPIC IX.	Fractional Equations
TOPIC X.	Powers, Roots, and Radicals
TOPIC XI.	Quadratic Equations Solved by Factoring
TOPIC XII.	Ratio—Basic, Proportion—Basic, and Variation

Topic I. Orientation

Review different types of communication, such as: gestures, deaf and dumb language, spoken language, telephone, radio, chemists' use of symbols for elements.

Develop the idea that symbols are used to express the elements of written language. Algebra is a "shorthand" developed to express mathematical relationships.

Point out that in ordinary language there are symbols that express general concepts and others that express definite, particular concepts.

In like manner there are, in the language of numbers, symbols that represent general concepts.

Topic II. Formulas

See section on Individualizing Instruction in Algebra, page 108, and Topic VII of Basic Mathematics I, page 114.

The formula is a shorthand rule; formulas are a specialized form of equations.

The letters of a formula may represent any number.

Formulas can be made from verbal statements, diagrams, and tables. Solve formulas by numerical substitution, especially for dependent variables such as in $A = \frac{1}{2}bh$; find A when $b = 6$, $h = x$. Develop the idea of relationship—*i.e.*, area dependent upon base, height. Develop the meaning of equality.

Topic III. Directed Numbers and Fundamental Operations (Exclude polynomials as divisors.)

See developed outline on page 198.

The meaning and use of numbers with signs[1]
Addition of numbers with signs
Addition of polynomials
Subtraction of numbers with signs
Subtraction of polynomials
Multiplication of numbers with signs
The sign of a power

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[1] The expression "signed numbers" is purposely omitted to avoid the confusion that often arises in e minds of pupils who conceive the idea that numbers with signs are a special category of umbers.

Division of numbers with signs
Division in fractional form
Multiplication of a polynomial by a number with a sign
Removing symbols of grouping in equations
How to write an algebraic equation interpreting a situation

Topic IV. Graphs

Be able to plot points, to graph a linear equation, and to interpret the graphs.

Topic V. Linear Equations in One Unknown (See also Chapter III, section on Enrichment)

See Suggestions for Problem-solving on page 198.

The use of fundamental operations
Finding the value of the unknown
The laws of the equation—axioms
Solve formula for independent variable such as: $A = \frac{1}{2}bh$, solve for b, solve for h

Topic VI. Linear Sets of Equations in Two Unknowns

Solving the equations by addition or subtraction
Solving by substitution
Solution by graphs

Topic VII. Special Products and Factoring

Understanding the meaning of factoring; also the meaning of squaring and of finding square root
Expressions containing a common monomial factor
The difference of two squares
General quadratic trinomials
Trinomial squares

Topic VIII. Operations with Fractions

Reducing to lowest terms
Expressing as integral or mixed expressions
Addition, subtraction, multiplication, and division of fractions
The sign of the fraction

Topic IX. Fractional Equations

Adding and subtracting fractions in equations
An understanding of the principles of clearing equations of fractio
Solving literal fractional equations

Topic X. Powers, Roots, and Radicals

The meaning of powers
Square monomials
The meaning of square root and the symbol for square root

Finding the exact square root of a monomial
Simplifying radicals (square roots)
Using tables of square roots
Simplifying and combining radicals (square roots)

Topic XI. Quadratic Equations Solved by Factoring

Topic XII. Ratio, Proportion, and Variation

An understanding of simple variation and of direct variation
Writing formulas and building tables illustrating direct variation
Developing the idea of ratio and using it in a proportion. (Show that both have extensive use in mathematics such as trigonometry and in practical applications—scale drawings.)

SUGGESTIONS FOR GUIDANCE IN SOLVING WORD PROBLEMS[1]

Since the solution of word problems is one of the main objectives sought in the study of algebra, and since it is also the most difficult part of algebra for the average child to understand, the study of word problems should receive much attention and emphasis.

The following suggestions for solution of word problems are recommended:[2]

1. Read the whole problem very carefully and try to understand its meaning thoroughly.
2. Whenever possible draw a rough figure or diagram.
3. Make a list of the given numbers.
4. Represent the quantity you are to find by x or by the initial letter.
5. If you are to find more than one quantity, represent the smallest one by x, and express the related quantities in terms of x.
6. From the conditions in the problem find two expressions that are equal.
7. Express as an equation that which the problem states in words.
8. Solve the equation for the unknown.
9. Solve for any other required values.
10. Check in the original problem by substituting the values obtained. Checking in the equation does not check the problem as the equation may be wrong.
11. If two or more unknowns are used, represent one unknown by x, the other by y, etc., or use the initial letters, then follow the same procedure.

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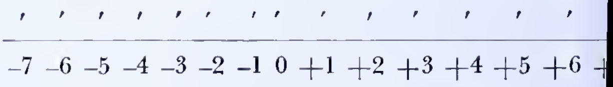
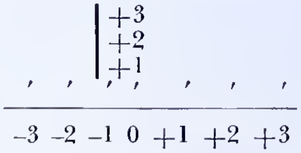
[1] Excerpt from *Curriculum Bulletin Number 531*, Orange Public Schools, Orange, Texas, 1946.
[2] Attention should also be given to developing the habit of estimating results and of checking the reasonableness of answers.

SUGGESTIONS FOR PROCEDURE IN TOPIC III

The following is Topic III with the subject content broken into subareas and including some suggestions for procedure. Teachers should adapt the suggestions of this unit and thus build the other topics in a manner which will give the best aid to the algebra teacher and his pupils.

SUBJECT CONTENT	SUGGESTIONS FOR PROCEDURE
III. Discovering a new kind of number, using directed numbers in formulas, equations, and problems	III. Introduce numbers with signs by using concrete illustrations of interest to the pupil. Review the number system as used in arithmetic (whole numbers, fractions, decimals) Create a need in the minds of the pupils for an extension of the number system by use of questions about the thermometer, elevator movements, battery charges, etc.
A. *The meaning and use of numbers with signs*	A. A negative number is a natural number preceded by a minus sign. *Example:* Profits and losses; latitude and longitude; position of an airplane as compared with that of submarine; show by means of the following diagram that the number system includes negative numbers.  Zero is used as an arbitrary starting point or the point of beginning. These numbers represent *distance* and *direction* from the point of beginning. Draw a vertical line and similarly indicate the number scale on it. Both scales may be combined as follows:  Teach the fact that mathematicians agree that the absence of a sign before a number indicates that it is to be considered a positive number. Be sure to let the pupils have many experiences working with numbers with signs.
B. *Adding numbers with signs*	B. Use the algebraic scale to find the sum of signed numbers, then state the rule.

Review the vocabulary of addition in arithmetic, *i.e., addend, sum.*

Suggest that adding may be considered as combining or taking together.

Review the number scales.

By means of the number scale, teach how to find the sum of two numbers with all possible sign combinations.

Teach how to find the sums of three or more numbers and include problems in which negative units neutralize a corresponding number of positive units.

Give practice in horizontal as well as vertical addition.

Review the meanings of *coefficient, factor* (numerical and literal), *exponent,* and similar terms.

Develop the addition of monomials with several factors as follows:

	Problem		*Check*
	$3 \times x = 3x$		$3 \times 6 = 18$
	$2 \times x = 2x$		$2 \times 6 = 12$
	$4 \times x = 4x$		$4 \times 6 = 24$
SUM	$9 \times x = 9x$	SUM	$9 \times 6 = 54$

Extend the idea of addition to negative monomials, to monomials with exponents.

C. *Addition of Polynomials*

C. Teach the meaning of the expressions: similar terms, combine, monomial, binomial, trinomial, and polynomial. Be sure that similar terms fall in vertical columns. Do not check by substitution of numerical values.

Review the idea that $3a + 2b$ means 3 times a number added to 2 times another number.

Show that adding columns of polynomials is similar to adding columns of monomials.

Point out the difference between a coefficient of 1 and 1 as a constant term.

Emphasize the point that only similar terms may be added.

Show that the addition of dissimilar terms may be indicated only.

Point out that polynomials are usually arranged in alphabetical order or in ascending or descending order of exponents of one of the letters.

Give practice with integral, fractional, and decimal coefficients, and with the terms arranged horizontally as well as vertically.

SUBJECT CONTENT

SUGGESTIONS FOR PROCEDURE

D. *Subtraction of Numbers with Signs*

D. Teach subtraction and develop the rule by the use of illustrations such as: the number scale, thermometer, money, etc. Check by addition.

Review the terms *minuend, subtrahend,* and *difference.* Analyze with the process of subtraction in arithmetic.

Lead the class to see that when we take 6 from 10 we find what is left when 6 units are taken from 10 units.

Show that subtraction may also be defined as the process of finding what number added to 6 gives 10, for it gives the same number as finding what is left when 6 is taken from 10.

Extend the arithmetic concept to numbers with signs, that subtraction may be considered as the process of finding what number added to the subtrahend gives the minuend.

After the process is thoroughly understood and mastered, develop with the pupils the fact that the same result may be obtained if the sign of the subtrahend is changed and the problem treated as one of addition rather than subtraction.

Extend the idea to polynomials.

E. *Subtraction of Polynomials*

E. Emphasize the importance of placing similar terms in vertical columns. Use the rule of subtraction.

Extend the ideas developed for numbers with signs to polynomials.

F. *Multiplying Numbers with Signs*

F. Correlate multiplication with addition of like addends. Develop the rules for multiplication of numbers with signs by the use of practical illustrations.

As an explanation of the rules for multiplying numbers with signs, the following two series may be helpful:

5	5	5	5	5	5	5
3	2	1	0	–1	–2	–3
15	10	5	0	–5	–10	–15

and

–5	–5	–5	–5	–5	–5	–5
3	2	1	0	–1	–2	–3
–15	–10	–5	0	+5	+10	+15

Assuming that the laws of increase and decrease are continuous as the multiplier passes through zero, the conclusions that (–3) × 5 = –15 and (–3) × (–5) = 15 are reached.

SUBJECT CONTENT	SUGGESTIONS FOR PROCEDURE
	Review the meaning of *multiplication, multiplier,* and *product.* Review the idea that where no coefficient is expressed, the coefficient is understood to be 1. Review the idea that where an exponent is not expressed the exponent is understood to be 1. Develop with the class the law concerning exponents in the process of multiplication. Stress the fact the we add exponents only when the bases are the same. Extend the multiplication of terms to include coefficients. Develop the different ways of indicating multiplication. Teach the squaring of a monomial. After the class has mastered the other types, teach the type involving numerical bases. Extend multiplication of monomials to include negative coefficients.
G. *The Sign of a Power*	G. Develop the rules through the process of multiplication: Any power of a positive number is positive. Any even power of a negative number is positive. Any odd power of a negative number is negative.
H. *Dividing Numbers With Signs*	H. Review the meaning of *dividend, divisor,* and *quotient.* By means of examples in arithmetic develop with the class the principle that the quotient times the divisor equals the dividend. Show that since this is true, the same law of signs must apply to division as applies to multiplication. The following rules should be firmly fixed in the pupil's mind through usage: When the dividend and divisor have like signs, the quotient is always positive. When the dividend and divisor have unlike signs, the quotient is always negative. The rules applying to coefficients and exponents in the process of division should be developed in a manner similar to that recommended under multiplication.
I. *Division in Fractional Form*	I. An expression such as $\frac{3a}{6b}$ should always be read as "3*a divided* by 6*b*."
J. *Multiplication of a Polynomial by a Number with a Sign*	J. Multiply every term of the polynomial by the number with signs, and follow the rules for multiplication.

SUBJECT CONTENT	SUGGESTIONS FOR PROCEDURE
	This topic may be introduced by showing that the process of multiplication in algebra is a generalization of the process of multiplication in arithmetic. Limit the degree of difficulty to binomials multiplied by binomials.
K. *Removing Parentheses in Equations*	K. When a quantity is preceded by a plus sign, the parentheses may be removed without changing the sign of any term within the parentheses. When the quantity in the parentheses is preceded by a minus sign, the parentheses may be removed if the sign of every term within the parentheses is changed. Review the use of plus and minus signs as signs of operation. Lead the pupils to see that parentheses are used to indicate an aggregation. Focus the attention of pupils on the fact that a minus sign preceding the parentheses is used as a sign of operation meaning subtraction, and that the expression within the parentheses is treated as a subtrahend. Have the pupils change the signs of each term in the subtrahend and explain to them that, having changed the signs, the subtraction has been done and the minus sign which was merely used to mean subtraction no longer functions. Make clear to the pupils that an expression enclosed in parentheses represents a single expression or concept. Explain to the pupils that the reverse process is also true, namely, that inserting the expression into parentheses preceded by a minus sign means a change of signs for each term of the expression. Follow the same procedure for teaching the removal of parentheses preceded by a plus sign. Show the pupils the different signs of aggregation.
L. *How to write the Equation of a Word Problem*	L. Require the pupils to read the problem carefully and analyze every sentence. (See section on Studying Mathematics, page 41.) Helen has $7.00, consisting of dimes and nickels. She has three times as many dimes as nickels. How many nickels and how many dimes does she have?

ALGEBRA II

GRADE TEN

THEME: *Mathematics and the Earner*

ATTENTION TO BASIC MATHEMATICS

No uniformity of thought has as yet been reached with reference to the exact placement of the second year of algebra, which is sometimes referred to as intermediate algebra. However, the committee considers it advisable to place this course in the tenth year.

Second-year Algebra may be elected by pupils wishing to meet the requirements for college entrance or for entrance to other higher institutions of learning, and by pupils not expecting to attend college but intending to engage in technical or semitechnical work or in commerce; other pupils may elect the course because they enjoy the subject, and still others to meet graduation requirements. Further information as to who should study second-year algebra may be obtained from the "Guidance Pamphlet in Mathematics for High School Students" prepared by the Commission on Post-War Plans of the National Council of Teachers of Mathematics.

Every teacher of second-year algebra should be familiar with the recommendations in the Third Yearbook of the National Council of Teachers of Mathematics. Articles appearing in the *Mathematics Teacher* and the treatment of special topics as given in certain recent textbooks will also be found in the teaching of the algebra at this level.

Basic Mathematics. It should be emphasized that the essence of this guide is for all pupils to have training in the common learnings of mathematics. In a specialized course, such as this, the teacher should give attention to the mathematics for daily living by:

1. Studying and utilizing the outline for the Basic Mathematics suggested for this same grade level
2. Emphasizing practical application of the topics in algebra
3. Stressing particularly the understandings in those topics marked "Basic" in the Algebra outline, p. 204. Such topics have comparable areas in Basic Mathematics. The essential difference is that in Algebra the concepts and techniques will be extended further than will be done in the similar topic as presented in the Common Learnings Program.

OBJECTIVES OF ALGEBRA II

The following objectives of Algebra II are the goals for pupil development, and therefore are guides for teachers in helping pupils to:

1. Develop a higher degree of skill and accuracy in algebraic techniques
2. Increase their power to solve equations with accuracy and reasonable speed
3. Acquire an understanding of the functional relationships between given facts
4. Strengthen their skill in developing equations to represent relationships between given facts
5. Acquire the ability to use algebraic concepts and techniques in such fields as the sciences, engineering, industry, etc.
6. Develop habits and attitudes which will be helpful in life situations
7. Increase their interest in and desire for further study of mathematics
8. Broaden their knowledge of the fundamental mathematical concepts relative to wage earning
9. Understand the general nature of algebra

OUTLINE OF ACTIVITIES BY TOPICS

TOPIC I. A Study of Relationships—Basic
TOPIC II. Fractions and Fractional Equations
TOPIC III. Review Graphs of First Degree Equations and Extend to Systems of First Degree Equations—Basic
TOPIC IV. Special Products and Factoring
TOPIC V. Powers, Roots—Basic, and Radicals
TOPIC VI. Graphs of Quadratic Equations
TOPIC VII. Quadratic Equations
TOPIC VIII. Logarithms
TOPIC IX. Series—Basic
TOPIC X. Binomial Theorem
TOPIC XI. Systems of Quadratic Equations

It is suggested that any necessary review of fundamentals be given throughout the course whenever and wherever the need arises rather than to begin the year with a review of fundamental operations.

Topic I. A Study of Relationships

Changes in related variables, variables and constants, accuracy in measurement, etc.

Topic II. Fractions and Fractional Equations

Reducing to lowest terms, multiplication and division, the signs of a fraction, addition and subtraction, reducing to mixed numbers, multiplication and division of mixed numbers, complex fractions, solution of fractional equations. (See more detailed outline on page 206.)

Topic III. Review Graphs of First Degree Equations and Extend to Systems of First Degree Equations

Pupils should be taught to recognize dependent and inconsistent sets or systems of equations.

Topic IV. Special Products and Factoring

The regular cases should be understood. The pupil should continue to factor until all factors are prime.

Topic V. Powers, Roots, and Radicals

Review the laws of exponents and extend to the meaning of zero, negative, and fractional exponents. Understand the meaning and use of square root. Pupils should be able to simplify, add, subtract, multiply, and divide radicals with reasonable speed and accuracy. The rationalization of denominators should be limited to denominators having two terms. Consider only radical equations which lead to linear equations.

Topic VI. Graphs of Quadratic Equations

Be sure to include equations that produce the circle, the ellipse, the parabola, and the hyperbola. Explain why these figures are called conics.

Topic VII. Quadratic Equations

Concepts to be developed are: incomplete quadratic, quadratic formula. Include quadratic equations with literal coefficients and equations involving radicals.

Topic VIII. Logarithms

Reveiw the laws of exponents for multiplication, division, powers, and roots. Show the related notations of exponents, radicals, and logarithms.

Topic IX. Series

The progressions offer the teacher a rich opportunity to link up his mathematics with important social, economic, and biological situations. For example, show that the reproduction of micro-organisms studied in biology follows a geometric progression. This is important in personal as well as in public health.

Topic X. Binomial Theorem

In no case should the pupils ever be required to memorize the formula.

Topic XI. Systems of Quadratic Equations

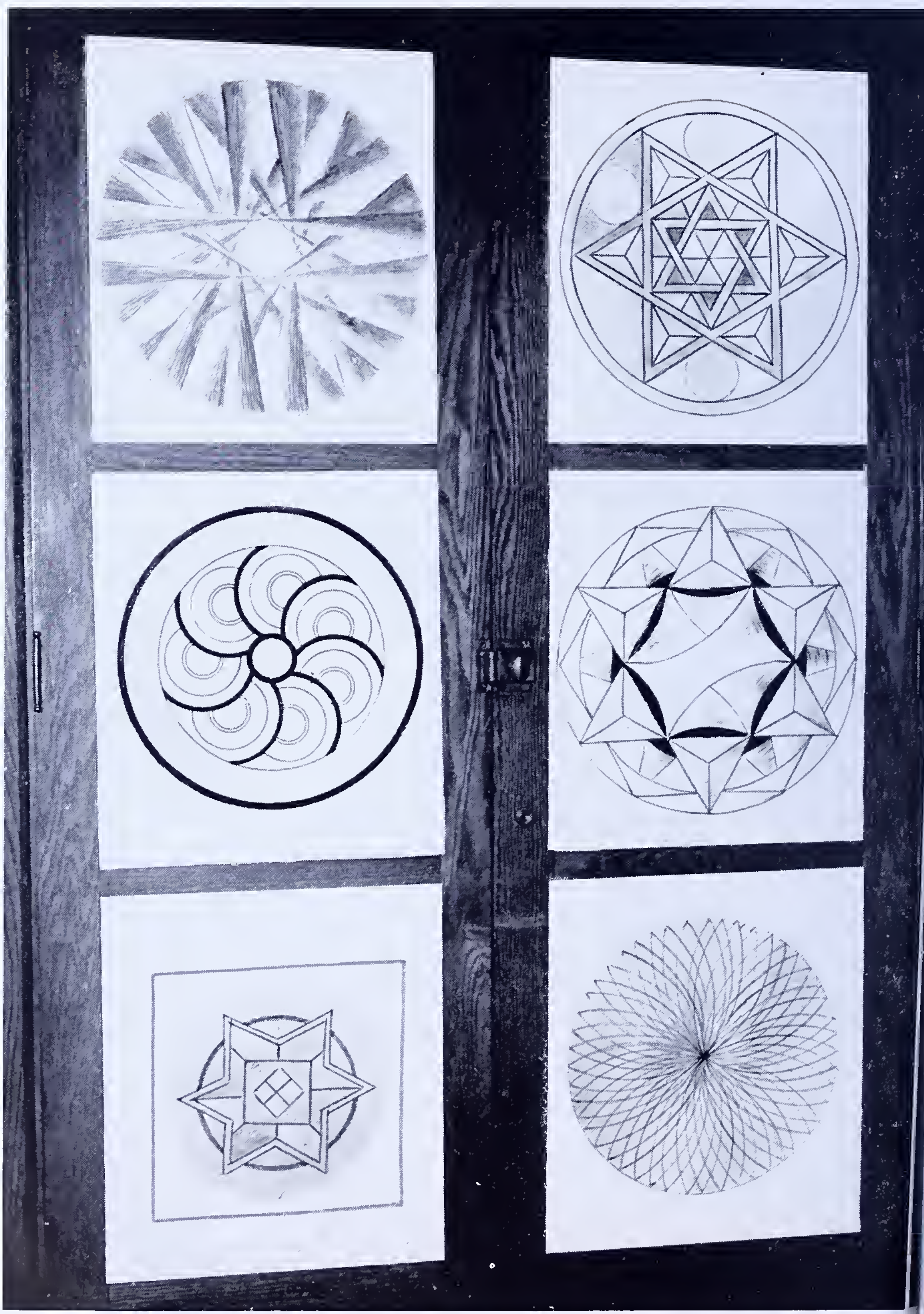
The addition and subtraction method and the substitution method will be used in the solution of systems of quadratic equations. Point out the similarity of this method and the corresponding one relating to linear equations.

SUGGESTIONS FOR PROCEDURE IN TOPIC II

Topic II with the content broken into subareas and with some suggestions for procedure. Teachers may adapt the suggestions of this unit in developing other topics.

SUBJECT CONTENT	SUGGESTIONS FOR PROCEDURE
II. *Fractions and Fractional Equations*	II. Use arithmetic to introduce every new algebraic manipulation to recall the pupils' past experience. Then correlate algebra with these experiences. The pupil should understand the basic principle of fractions, namely, the value of a fraction is not changed if both numerator and denominator are multiplied or divided by the same number (not zero). Correlate with multiplication and division of fractions in arithmetic. Multiply fractions as follows: Reduce each integral or mixed expression to fractional form. Factor every polynomial which is not a prime factor. Divide the numerator and the denominator by the common factors. Multiply the remaining terms in the numerator for the numerator of the product and the remaining factors in the denominator for the denominator of the product. Pupils should understand that 1. Factors, not terms, are eliminated by division. 2. When all the factors in the numerator have been eliminated by division, the value of the numerator is 1. 3. When all the factors in both numerator and denominator have been eliminated by division, the result is 1, not zero. See note in "Do's and Don'ts," page 26.
A. *How the Value of a Fraction Changes*	A. Introduce variation; show the relation between variables. Explain that the value of an algebraic fraction depends upon the values of the letters in it and that the value usually changes when the values of the literal numbers in the fraction change. Explain the meaning of proper and improper fractions in algebra and in arithmetic.
B. *The Sign of a Fraction*	B. Emphasize the three signs of a fraction and ways for changing any of them. There are two general cases that require treatment:

SUBJECT CONTENT	SUGGESTIONS FOR PROCEDURE
	1. When the terms of the fraction are not in fraction form. 2. When the terms of the fraction are in fraction form. Stress the difference between changing the signs of an expression and rearranging the terms of an expression.
C. *Addition and Subtraction of Fractions*	C. Before introducing addition and subtraction of fractions, practice finding the lowest common multiple of polynomials. Then proceed as in arithmetic.
D. *Reducing Fractions to Mixed Numbers and Mixed Numbers to Fractions*	D. Reduce fractions to integral or mixed expressions by division. Here the corresponding rule in arithmetic may be applied. Note that in arithmetic when a sign is omitted between a whole number and a fraction, the plus sign is usually understood but in algebra a multiplication sign is understood.
E. *Multiplication and Division of Mixed Expressions*	E. Change the mixed expressions to simple fractions. Factor the polynomials when necessary and then multiply or divide.
F. *Complex Fractions*	F. A complex fraction is another way of expressing division and may be simplified as a division problem. Some complex fractions may be simplified by a shorter method, i.e., by multiplying both numerator and denominator by the same quantity. Two methods should be pointed out: 1. Reduce numerator and denominator to simple fractions and divide. 2. Multiply both numerator and denominator by the L.C.M. of all the denominators. Show the pupils that complex fractions may be written either in the vertical or horizontal form.
G. *The Solution of Fractional Equations and Problems Involving Fractional Equations*	G. Where polynomial denominators occur, factor them if possible. Find the L.C.M. of the denominators of the fractions and multiply each fraction and each integral term of the equation by it. Then solve as usual. Simplify when possible. Literal fractional equations will be of value in preparing for the solution of formulas involving fractions.



PUPILS FIND GEOMETRY IN DESIGNS

GEOMETRY

GRADE ELEVEN

THEME: *Mathematics for the Citizen*

ATTENTION TO BASIC MATHEMATICS

The organization of a course in plane geometry in grade eleven depends greatly upon the thoroughness with which the informal geometry was handled in the previous grades. Many of the important facts of geometry can be learned by the pupils through their experiences with informal geometry before reaching grade eleven. The study of informal geometry should also furnish a substantial beginning vocabulary of geometric terms. If these objectives have been attained through informal geometry in the previous grades, the formal geometry course can devote itself largely to studying the logical organization of the materials of geometry.

The significance of defined terms, undefined terms, assumption, and previously proved theorems is of exceptional importance. The logical demonstrations of theorems should be held forth as patterns of thinking that must be followed in all fields of scientific research. Nowhere in the school curriculum is there a more appropriate place to have the students learn about the nature of proof. This study of proof should not be limited to mathematical materials but should be extended into many of life's areas outside geometry.

A unit on "Ways of Reaching Conclusions in Life" might be an excellent approach to the study of the roles played by intuition, induction, and deduction in the thought processes of life. Topic VIII below on "The Nature of Proof" presents some suggestions which might form the basis for such a unit.

The number of theorems in plane geometry may be reduced by postulating many of the theorems now frequently demonstrated or proved. For example, the theorem concerned with vertical angles, the angles formed by parallel lines cut by a transversal, and the congruence theorems should all be assumed in demonstrative geometry. The beginning teacher often has difficulty in knowing where to put the proper emphasis on the many theorems found in some texts. The following list of 16 theorems[1] will give the pupil sufficient tools to do 60 per cent of all exercises of a geometric nature.

Theorems

1. In a right triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.
2. The sum of the angles of a triangle is equal to two right angles.

---

[1] Doctoral investigation of Dr. Hale Pickett, West Chester State Teachers College.

3. An angle inscribed in a circle is equal to half the central angle having the same arc.
4. The area of a triangle is equal to one-half its base times its altitude.
5. If two sides of a triangle are equal, the angles opposite these sides are equal.
6. Two triangles are similar if the two angles of one equal respectively the two angles of the other.
7. The area of a circle is πr^2.
8. Two triangles are congruent if two sides and the included angle of the one are equal, respectively, to two sides and the included angle of the other.
9. The areas of two similar triangles (polygons) are to each other as the squares of any two corresponding sides.
10. When a transversal cuts two parallel lines, the alternate interior angles are equal.
11. If a straight line is drawn through two sides of a triangle parallel to the third side, it divides these sides proportionally.
12. In any circle, angles at the center are proportional to their intercepted arcs.
13. In any parallelogram the opposite sides are equal.
14. In a 30° to 60° triangle the side opposite the 30° angle is equal to one-half the hypotenuse.
15. In any right triangle the perpendicular from the vertex of the right angle on the hypotenuse divides the triangle into two triangles, each similar to the given triangle.
16. Two right triangles are congruent if the hypotenuse and one other side of one are equal, respectively, to the hypotenuse and another side of the other.

Since people live in a three-dimensional world, the practical facts, or aspects, of solid geometry should be handled on an informal basis with the more formal work in plane geometry. Thus, most of the important facts and the vocabulary of solid geometry would be presented in grade eleven along with plane geometry. For example, when the locus producing a circle is studied, the corresponding locus that produces a sphere would be considered. Much practice should be provided to interpret three dimensional drawings placed on a two-dimensional surface. Pupils must expand their concepts beyond the "geometry of flatland."

Basic Mathematics. It should be emphasized that the essence of this guide is for *all* pupils to have training in the common learnings of mathematics. In a specialized course, such as geometry, the teacher should give attention to the mathematics for daily living by:

1. Studying and utilizing the outline for the Basic Mathematics suggested for this same grade level.

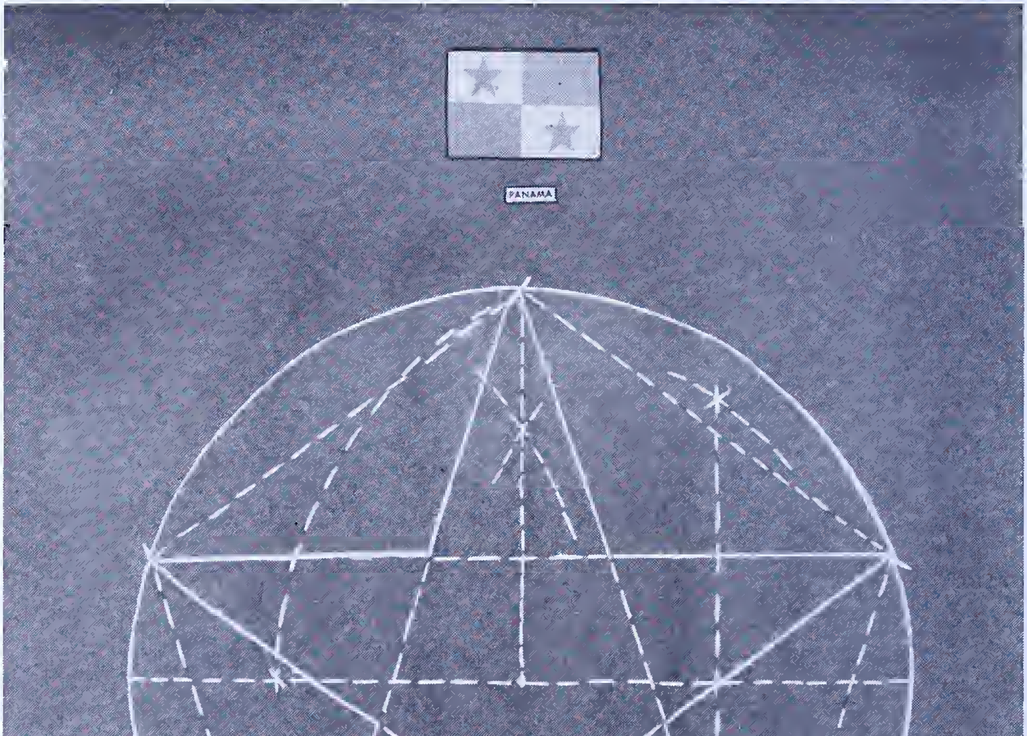
2. Stressing particularly the understandings in those topics marked "Basic" in the Geometry Outline, p. 213. Such topics have comparable areas in Basic Mathematics. The essential difference is that in geometry the concepts and techniques will be extended further than will be done with the similar topic as presented in the Common Learnings Program.
3. Emphasizing practical applications of the topics in geometry.

OBJECTIVES OF GEOMETRY

The following objectives of geometry are the goals for pupil development and therefore are guides for teachers in helping pupils to:

1. Understand the relationships of geometry to arithmetic and algebra
2. Increase their knowledge of geometric forms and develop an understanding of geometric concepts
3. Know the principles, standards, and processes of objective thinking in order that there may be developed the ability to think critically on problems of life as well as on problems in geometry
4. Know and appreciate fundamental geometric propositions, as well as understand geometric terminology
5. Appreciate the significance of geometry as it applies to science, engineering, art, and other fields
6. Develop the habit of defining terms, thinking accurately, and establishing conclusions both geometric and non-geometric realizing that
   a. Undefined terms exist in every body of knowledge
   b. The axioms and postulates of mathematics are really assumptions
   c. Basic assumptions (stated or unstated) have a direct bearing on the conclusions reached
7. Further their concepts, interests, and appreciations in more extensive geometric study where the aptitude exists for such study
8. Acquire a concept of spatial relationships including those of three dimensions

This course, in other words, seeks to develop a mathematical maturity rather than the mere acquisition of the details of geometry.


PANAMA
PENTAGON

OUTLINE OF TOPICAL DEVELOPMENT

TOPIC I. Introduction
TOPIC II. Fundamental Concepts—*Basic*[1]
TOPIC III. The Working Tools in Geometry—*Basic*
TOPIC IV. Constructions—*Basic*
TOPIC V. Angles
TOPIC VI. Triangles
TOPIC VII. Congruence
TOPIC VIII. The Nature of Proof—*Basic*
TOPIC IX. Parallels and Perpendiculars
TOPIC X. Converse Theorems
TOPIC XI. The Quadrilateral Family
TOPIC XII. Polygons
TOPIC XIII. Circles
TOPIC XIV. Locus
TOPIC XV. Loci in Construction Problems
TOPIC XVI. Ratio and Proportion—*Basic*
TOPIC XVII. Correlation of Ratio and Proportion with Triangles and with Parallel Lines
TOPIC XVIII. Similarity—*Basic*
TOPIC XIX. Indirect Measurement
TOPIC XX. Equivalent Figures—Areas and Volumes—*Basic*
TOPIC XXI. Inequalities
TOPIC XXII. Inscribed and Circumscribed Regular Polygons

Topic I. Introduction

There has been a feeling on the part of geometry teachers for some time that geometry needs a drastic revision in its methods and approach. The teacher confronted with the problem of presenting a course in geometry is likely to feel that the standard textbook's method is the only method or infer that, since most texts follow the same pattern, it is the best one. Often an excellent job of teaching can be done without using a textbook at all. For the first month, especially, textbooks should not be used regularly. Even later on, there is the danger that too close adherence to the printed page will result in rote learning. Understanding in geometry should be developmental, not just memorization. However, a number of standard and experimental textbooks should be available in the classroom for frequent use and reference.

As an introduction, the teacher might mention to the class that many of the facts which are to be studied will be based on figures with which they are already familiar, such as the square, the rectangle, and the circle, then let the class start to list[1] all the items in their immediate environment involving geometric figures which they recognize.

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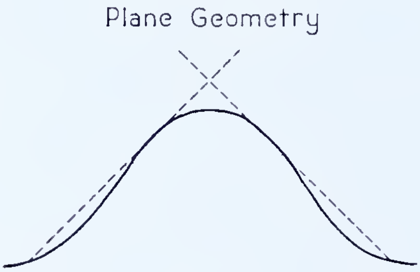
[1] The fundamental concepts of nearly every topic listed are of value to all youth, but those marked *Basic* should be particularly stressed.

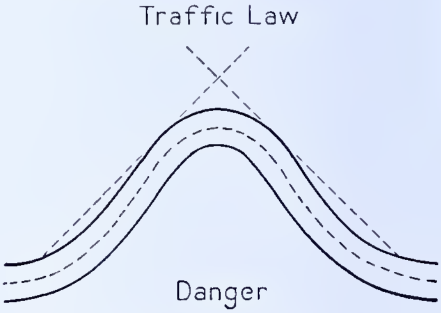
SUGGESTED FORM FOR A NOTEBOOK WORK SHEET

A. Geometric applications in pupil's life, pictures, industrial uses

The pupils' immediate environment should be a constant source of illustrations and applications of the topics being studied.



The "line of sight" is a straight line from the eye of the observer to the object being observed.



Do not pass a car going in the same direction on the brow of a hill. Do not pass on a curve.

B. Definitions

These should be the result of the pupils' discovery and should be written in his own words. The importance of definition can be brought out by talking with pupils about matters of their own experience—"a foul ball," "sophomore," "forward pass." The teacher should then begin to help the pupils to distinguish between geometric statements which are definitional and those which are propositional. For example, in the case of an isosceles triangle, such statements as "the base angles are equal" and "the altitude bisects the base" are *propositional.* The *definition* of an isosceles triangle refers only to the equality of two sides.

C. Constructions

This section will eventually include demonstrations of such items as: Reproducing a line segment, reproducing an angle, bisecting a line, bisecting an angle, constructing a perpendicular, constructing triangles, constructing parallels.

Topic II. Fundamental Concepts: Point, Line, Circle

Topic II should be devoted to having pupils appreciate the fact that all the space relationships which they will study in geometry will grow out of three basic elements: the point, the line, and the circle.

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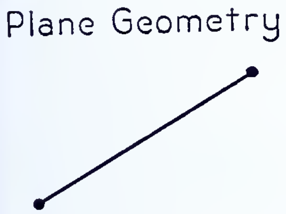
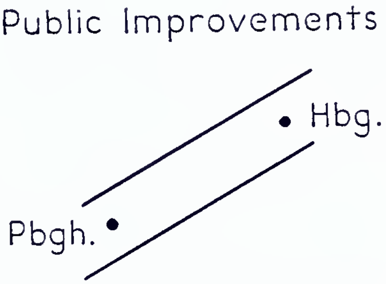
[1] Notebooks or pupil-developed workbooks are highly desirable for each member of the class.

The remainder of the topic should be devoted to working with the pupils to develop definitions of these basic elements. Note that these definitions should grow out of class discussion and decisions on what defines a point, a line, a circle. Here a start should be made on carrying two dimensional concepts into the third dimension.



A straight line is the shortest distance between two points.



The "Dream Highway" with its seven tunnels cutting through the mountains provides a short route from Pittsburgh to Harrisburg. The Liberty Tunnels in Pittsburgh use this principle to save valuable time for motorists.

Topic III. The Working Tools in Geometry

Introduce the tools which will be used in the study of geometry. Pictures, sketches, definitions, and manipulative experiences with these provide the learning activity.

Topic IV. Constructions

Continue the previous topic. The pupil now has an opportunity to experiment in producing with his geometric tools those three simple figures which he has already defined.

He should produce straight lines, and distinguish between straight lines and line segments. He should produce circles and consider what determines them. Much discussion will arise. The student will be surprised to discover how many parts of the circle he already recognizes—arcs, radius, diameter, etc. These terms may arise from pupil suggestions.

Topic V. Angles

Here the teacher might expect to introduce such concepts as the combination of two of the elements taught before. This will lead quite naturally to the discussion of angles. The most familiar thing in the student's life is his name; as a simple illustration point out that he has a family name and then a further identifying first name. "Angle" is a family name and the different kinds correspond to first names. The teacher will have to recognize that while the students will be able to recognize and identify some of these angles, some will have to be identified and classified by the teacher.

As a further suggestion, it is recommended that these should be taught and always presented in the following sequence: acute angle, right angle, obtuse angle, straight angle, reflex angle.

In order that the pupil visualize an angle in the natural position for an angle in the first quadrant in trigonometry, it is desirable always to draw angles on the board when they are being presented for class consideration in the following way:

The first geometric figure consists of two intersecting straight lines. This provides a point of beginning for study of the meaning of adjacent, supplementary, and complementary angles.

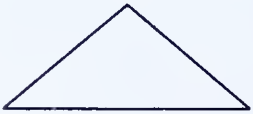
The next step is to use line segments to determine a single angle, name its parts, and label it correctly. The methodology of correctly naming an angle should be a point of emphasis here.

This is an excellent place to introduce solid geometry concepts. The pupil should begin to see what relationships develop when he substitutes the word "line" for "plane" and "angle" for "dihedral angle" in the statements which he uses in plane geometry.

Review should be made of measurement of angles with a protractor. Draw upon applications from aviation, artillery fire, industry. Have pupils take bearings with simple field protractors or with a magnetic compass.

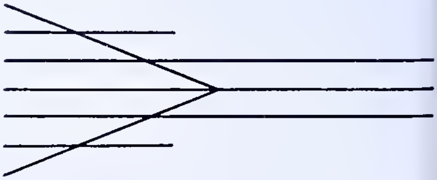
Topic VI. Triangles

Plane Geometry



An isosceles triangle is a triangle two sides of which are equal.

Public Improvements



An isosceles triangle is a safety factor in directing traffic as a highway changes from four lanes to two lanes.

Pupils should be encouraged to speculate on what would logically follow angles; triangles will doubtless be the outcome.

With this decided, refer to the family name illustration again, and note that triangles is the family name and that there are various "first names" such as isosceles, equilateral, acute, right, and obtuse.

Attention should be directed by the teacher to the fact that some of these are classified by angles and some by sides.

By the use of models show how different types of triangles, when revolved about certain sides, generate common solids.

Topic VII. Congruence

The study of congruence should involve construction of triangles and angles of various kinds; the pupils should be encouraged to identify them in every instance. A possible technique here might be to have students draw figures and exchange papers to see if the figures can be identified.

In this topic we are primarily concerned with having pupils determine what elements are necessary to reproduce a triangle. This will lead to the ASA, SAS, SSS, and AAS situations. From these arise the geometric statements which are classified as postulates. It is advisable to postulate the congruence theorems and to spend more time on originals which use these postulates.

As a further development of this same thesis, the right triangle situations of hypotenuse and acute angle, hypotenuse and leg can be introduced and worked. Lead the pupils to discover the various relationships as well as prove them.

Topic VIII. The Nature of Proof (in geometry; in daily living)

A. DEVELOPMENT OF FORMAL PROOF OF A THEOREM BY THE LABORATORY METHOD

In this development the base angles of an isosceles triangle are used.

Experience No. 1

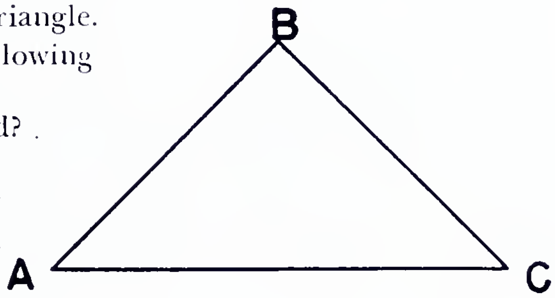
Here is a picture of a triangle.

Can you answer the following questions?

1. How is it labelled? .

____

____



2. What kind of triangle do you think it is? ____________
3. What would you do to make sure of this? ____________
4. What have you discovered? ____________
5. What kind of triangle is it? ____________
6. Measure angle *A* and angle *C* ____________
7. Measure side *AC* and side *BC*. ____________
8. In this space, draw a triangle that has two sides equal.
9. Measure angle *A* and angle *B* in this triangle. ____________
10. Could you draw a conclusion from this experience? ______

Experience No. 2

In Experience No. 1, we found that whenever two sides of a triangle were equal, then the angles opposite those equal sides seemed to be equal. Let us now attack this problem and see if it can be proved purely by reasoning without any need for measuring.

1. What must we start with? ____________
2. Can you draw this? If so, put it here.
3. What things are equal? ____________
4. What things do you want to prove are equal? ____________
5. What have we already learned that would prove two angles are equal? ____________
6. This requires that we have ____________
7. What should we do to the figures we started with that would give this? ____________
8. How should this line be drawn? (This line is called an auxiliary line and should be explained and discussed in detail.) ____________
____
9. Now, what conclusion can you draw? ____________
10. How did you arrive at this conclusion? ____________
11. Write a statement now of what you have done. ____________
____
____
____

Experience No. 3

Now let us develop a clearer and more concise way of doing this. Let us put down a statement which contains the things we are given to work with and what we want to prove.

1. What is given? ____________
2. What is to be proved? ____________

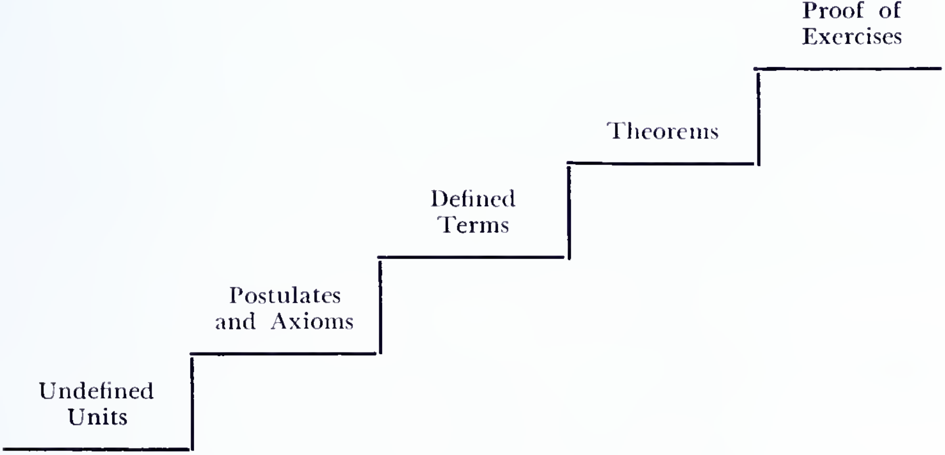
Now let us draw a picture of these conditions.

Next let us list step by step the things we did to prove this fact. We will call the statements of things we did "statements" and the reasons for doing them "reasons" and put them in parallel columns opposite each other, thus:

Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.

Experience No. 4

This experience should be devoted to solving an original exercise by methods of reasoning.



TO REACH THE TOP, TAKE THE STAIRS

B. THE DEVELOPMENT OF CRITICAL THINKING[1]

One of the major goals of secondary education is to develop an intelligent citizenry. All subjects should make a contribution to the growth of ability to think clearly, to solve problems of life, to reflect upon the nature of ideas and wants. It has long been a claim that the study of mathematics will develop ability to reason, but studies and observation on how we learn have indicated that automatic transfer of problem-solving ability from the study of pure mathematics to the problems of life is a controversial subject. However, when the study of mathematics is carefully organized and purposively taught for application to living, it offers excellent opportunities for presenting the basic processes of critical thinking. The "if-then" nature of geometry is particularly well suited for the development of critical thinking. Teachers of any subject, but teachers of mathematics in particular, may well build suitable learning experiences upon the following principles of good thinking:

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[1] Max Black, *Critical Thinking.* New York, Prentice-Hall, Inc., 1946.

1. Good thinking is free from emotion or bias and does not employ terms or names that carry an emotional impact.
2. Good thinking grows from a clear understanding of all aspects of the problem.
3. Good thinking develops from a recognition of the relevancy, adequacy, dependability, and organization of all data pertinent to the problem.
4. Good thinking must be free from hasty generalizations, false assumptions, false analogies, and irrelevant conclusions.
5. In good thinking, all conclusions are regarded as tentative or qualified until they have been proved.

C. STEPS IN LEARNING HOW TO THINK OBJECTIVELY

1. Understanding and recognizing a problem
2. Assembling and logically arranging all pertinent facts
3. Considering all possible solutions of a problem
4. Applying the most reasonable solution as a tentative solution
5. Establishing and judging claims of solution

D. SUGGESTIONS FOR INCORPORATING EACH STEP INTO THE TOPIC

1. Recognizing and understanding a problem

   a. *Recognizing assumptions*

   Most problems of life are predicated on certain assumptions that are either stated or implied. In understanding a problem these assumptions must be recognized and evaluated. In most mathematics texts the assumptions in verbal problems are often implied. The first step in recognizing these assumptions is to read over some word problems and ask the class for the implied or unwritten assumptions.

   Example: If four men working eight hours a day could complete a piece of work in twelve days, how long would it take six men to do the same work, working six hours a day?

   Assumption: All men do an equal amount of work in one day.

   b. *Emotional words*

   (1) Present to the class the following specimen of emotive writing:

   A fabulously rich playboy, who grew tired of his ponies, got the idea that he would like to repudiate the free enterprise that privileged his grandfather to endow him with so many many million dollars that he could never hope to count them.

(2) Have the class identify the emotive suggestions as follows:

LANGUAGE USED	SUGGESTION CONVEYED
"playboy," "ponies"	The man in question is an idler and gambler.

(3) Have the class discuss the meaning which the writer attempted to convey. In effect, it might be as follows:

The man in question is an idle gambler, who has far more money than he deserves, and is now irresponsibly using the vast financial power which he did nothing to earn.

(4) Have the class rewrite the statement with words having the opposite emotive tendency. In effect, it might read:

A very wealthy American sportsman has decided to oppose the system of unregulated commercial trading which enabled his grandfather to leave him his large fortune.

(5) Perform this exercise several times on biased writings which are found in locally available publications.

c. *Ambiguity*

(1) In developing the concept of ambiguity, take the simple word "man" and call attention to its many uses. How is the word used in each of the following?

(a) Man the lifeboats!
(b) The son of man
(c) He is only sixteen—not yet a man
(d) Man is an animal
(e) Get that man!

(2) Having demonstrated the principle of ambiguity, the teacher could present additional statements with ambiguous meaning and request the pupils to bring in some of their own findings or invention. In what different ways is the word "angle" used?

In this assignment the teacher could suggest words that may be ambiguous, such as:

Table
Education
Solid
School
True
Plane
Compass
Citizenship
Good
Right

(3) In summarizing the work the teacher should point out that words are, in effect, used to convey meaning. They can be used also to obscure the real meaning. One of the most common causes for arguments and misunder-

standings in life is the lack of common agreement on the precise meaning of words and terms used.

d. *Definition*

The need for definition arises automatically out of the use of emotionally tinged and ambiguous words in the statement of the facts and assumptions of any given problem. Consider the definition of words used so far in the course. Is there need to reword to avoid ambiguity in the following?

An article in a weekly news magazine began by saying that "Britain is actively at work in her plans for a Western European block of nations" and continued with a discussion of the apparent desire of Britain to form an alliance with France, Belgium, Denmark, Norway, and the Netherlands. Later in the article is the following:

> "British spokesmen admit that there are dangers in their plan and say they dislike the thought of bisecting Europe into two blocks."

When Helen, who was studying geometry, read that quotation, she said: "Oh! The man who wrote that made the same kind of mistake I did when I said that the line segment *PQ* was bisected into two parts. He ought to know better than that."

What do you think is the "mistake" to which Helen referred?

Rewrite the quotation so that it states clearly and unambiguously just what you think it means.

2. Assembling and logically arranging all pertinent facts

a. The teacher must take great pains to point out that all future action is based on a thorough objective understanding of the problem, and that once having reached such an understanding the next logical step is to ask: What must I know to solve the problem?

b. Having made this point clear, the teacher can then select word problems from the text and organize a class discussion around the theme, such as: What would be a good method of picking out the essential facts in these problems, and how can we organize them so that they fall into their proper order for consideration?

c. Having progressed to the point where pupils see the need for organization of facts, an illustration should be introduced that will contain data having no bearing on the problem.

Example: A black-and-white-tiled swimming pool 75 feet long and 36 feet wide is built 140 yards from a waterfall in Clearwater Brook. How far must Jack swim if he is to swim the length one and one-half times?

d. As an assignment, ask the pupils to bring in two problems containing irrelevant data. During the next class session they can be given the opportunity to evaluate each other's problems and to organize the data and pick out those which are pertinent. Extravagant advertising often uses irrelevant details to influence the buying public.

e. A group of people who were taking an automobile trip were discussing some of the causes of accidents. One man mentioned that many people are killed when cars are struck by railroad trains at crossings. One man said: "Assume the following facts are true: The flasher signals are operated by batteries, electrical circuits, switches, and other delicate mechanisms. You approach the crossing and the lights of the signal are out." Which one of the statements below then follows?

_____ A train may be approaching.
_____ A train is not approaching.
_____ No conclusion is possible.

Check the reasons that you feel support your conclusion:

_____ (1) The electrical circuits may be broken and a train may or may not be approaching.
_____ (2) One could only conclude that a train is not approaching.
_____ (3) The soundness of this kind of argument depends upon whether all the possibilities have been considered.
_____ (4) Railroad men say the thing to do is to look each way before crossing the track.
_____ (5) If we accept the assumptions on which an argument is based, we must accept the conclusion which follows logically from those assumptions.
_____ (6) "Delicate mechanisms" needs to be defined before a conclusion can be drawn.
_____ (7) The batteries may be worn out.
_____ (8) Such warning signals are apt to be unreliable and not much attention should be paid to them.

f. This aspect as integrated with the subject matter of the course

(1) The teacher should insist on some written organization of facts, as a matter of course, in the solution of verbal problems. The teacher should be certain that

the verbal problems, by nature, approximate real life conditions and contain information that does not have any bearing on the problem itself. He should do this so that the pupil may develop the ability to discriminate between data that are pertinent and data that are not.

(2) The teacher should encourage students to illustrate data in graphic form, or to develop some picture of the data. In this manner the pupil will learn to see how data are related and will also better understand the problem. Geometric problems, especially, lend themselves to a sketching of the given conditions.

3. Considering all possible solutions of the problem
   a. Induction and deduction are two important methods for seeking conclusions.
   b. Reaching a conclusion based on known data is the point at which many fallacies of reasoning occur. An effective tool for developing such an attitude is the syllogism, which is an argument composed of two premises and a conclusion.
      (1) The teacher may present the following line of reasoning to the class: (a) All men are human begins; (b) Mr. Jones is a man; (c) Thus, Mr. Jones is a human being.
      (2) The teacher points out that if statements (b) and (c) were reversed, (b) would be the necessary conclusion of (a) and (c), and that in the argument as presented (a), (b), and (c) represent a necessary conclusion.
      (3) As an assignment each student is asked to bring in two syllogisms, one with a valid conclusion, and one with an invalid conclusion. These are discussed in class.
   c. Conclusions by analogy are those which make a comparison between a given situation and another which bears some resemblance to the given situation.
      (1) The teacher should explain to the pupils what an analogy is and then, by returning to the syllogism, demonstrate the error in the analogy.
         (a) All great race horses are sired by champions.
         (b) The collie dog *Lassie* was sired by a champion.
         (c) The collie dog *Lassie* is a great race horse.
      (2) Pupils are asked to bring to class similar misleading analogies.

4. Applying the most reasonable solution as a tentative solution
   a. Writing the conclusion or solution to any problem is a very important aspect of objective thinking. The answer or con-

clusion should (1) pertain directly to the question asked or the problem stated, (2) not go beyond what the data require. Any part of the conclusion that is not completely supported by the data is speculation, and should be labeled as such. Speculation is important since it usually leads to further investigation; however, it is essential that the critical thinker be able to discriminate between a valid conclusion based on facts known to be true and suspicions or speculations based on incomplete data.

b. The teacher may write the word "conclusion" on the blackboard and divide the class into committees. Then after each committee has obtained a dictionary, each group can develop its own definition of the word "conclusion" based on the dictionary term and the group's knowledge of the process of objective reasoning.

5. Establishing and judging claims of solution

a. This is the point at which the actual technique of criticism is developed. Pupils now should have a concept of the process of deductive thinking and are ready to use this process in critical thinking. It is believed that a good method to bring this about is to have the class develop, cooperatively, a "check list" which they will call "Criteria for Objective Thinking."

b. The teacher places before the class the principles of objective thinking so far as the class has progressed. The teacher states, "In view of what we know, what are the things the objective thinker must not do?" The class may then develop criteria similar to the list below:

Criteria for Objective Thinking—

(1) Are the data or the conclusion influenced by prejudice or emotion?
(2) Is the problem (or issue) stated clearly?
(3) Are the data adequate?
(4) Are the data dependable?
(5) Are the data relevant?
(6) Is there evidence of "begging the question"?
(7) Is there evidence of "hasty generalization"?
(8) Is there any use of analogy? If so, is it misused?
(9) Has the conclusion been proved by actual application?
(10) Is the conclusion irrelevant?
(11) Is speculation or suspicion labeled as such?
(12) Does the conclusion contain "selling language"?
(13) Does the conclusion go beyond the data?
(14) Are the assumptions justified?

(15) In the deductive argument, is the source of the generalization shown?

c. Perhaps the best starting point for the teacher who has never before attempted to measure the ability of his pupils to think critically, is to use alternate forms of standardized tests. Form A should be given early in the course and Form B near the end to show the progress. Critical thinking is a relatively new field in so far as standardized testing is concerned, but there are several highly regarded tests on the market.[1]

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[1] Watson, Goodwin, and Glaser—*Watson-Glaser Tests on Critical Thinking.* World Book Co., Yonkers, N. Y. These tests have two forms IA and IIA.

Raths, Louis—"A Thinking Test." *Education Research Bulletin XXIII.* This is a test for the upper elementary grades although it could be used in the junior high school. The teacher with senior high school groups may get suggestions from the technique here employed so that he may develop appropriate activities for his own group.

Topic IX. Parallels and Perpendiculars

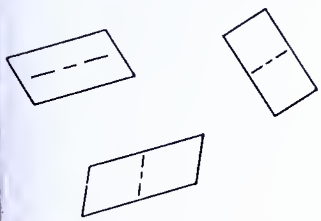
The teacher might introduce this topic by calling attention to such things in the common environment as lines of ceiling and floor of classroom, and railroad tracks. He may ask students to contribute from their own environment examples of lines which apparently do not meet. The pupils should explore this situation and come to the conclusion that these lines have a special name and are called parallel lines. From this, develop a third line that intersects them and its name "transversal." Use T-square, draftsman's triangles, and compasses for constructions.

Now the pupils should be directed to a project of finding all the possible combinations made by these three lines. This, then, should be further pointed up by calling their attention to the fact that some of these angles are between the parallels, some are outside the parallels, some are on the same side of the transversals, and some are on the opposite side of the transversal. At this point the teacher can readily see that it will be necessary to assist the class in discovering some first names of this family of angles. This will quite naturally lead into such facts as: (1) that the alternate interior, the alternate exterior, and the exterior-interior angles are equal; and (2) that the interior and exterior angles on the same side of the transversal are supplementary.

At this time, the theorem, "If two lines in the same plane are perpendicular to the same line, they are parallel," and further perpendicular theorems should be developed.

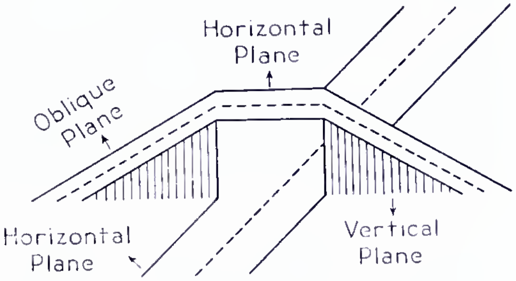
Here is an excellent place to bring in concepts of solid geometry by showing how this principle can be carried into three dimensions.

Plane Geometry



Lines which cannot meet, yet are not parallel, lie in different planes and are called skew lines.

Public Improvements



The overpass or the underpass provides additional safety for cross traffic over railroads or busy highways.

Topic X. Converse Theorems

This topic provides an excellent opportunity to introduce theorems and their converses. The class might begin with the teacher's presenting such examples as:

> **According to the Constitution, all presidents must be American citizens by birth. Abraham Lincoln was a president and therefore he was an American-born citizen.**
>
> **All fish swim through water. John saw something swimming through the water; therefore he knew it was a fish.**

At this point, it might be well to have the pupil draw a line to serve as a transversal; select on this line two separate points and at these points make the alternate interior angles equal. Now extend the sides of these angles in both directions. The pupils should now be called upon to decide what sort of lines are the result. Then the pupils should draw two parallel lines, cut by a transversal, and measure the alternate interior angles; again the class should be required to formulate some decisions concerning the angles. At this point the class should be in a position to formulate an opinion concerning parallel lines cut by a transversal and they should be encouraged to record in their notebooks the theorem and its converse which evolved. The pupil should be encouraged to utilize knowledge previously acquired to build the additional parallel theorems and converses.

Topic XI. The Quadrilateral Family

The pupil's attention should be directed to the fact that the knowledge he has been acquiring has dealt with consistent, logical, and orderly development of figures: first of two lines, then three lines, and now four lines, namely the quadrilateral. Pupils should be encouraged to construct four-sided figures of different shapes. The natural outgrowth of this will lead to using two pairs of parallel lines to determine a quadrilateral. This leads quite naturally to parallelograms. Further developed parallelograms will lead to such figures as rhombus, rectangle, square. Two, and only two, pairs of parallel lines define a special case of a quadrilateral, namely the trapezoid and its special case, the isosceles trapezoid. Show that these plane figures serve as bases and faces of prisms.

In practical applications, stress the uses of the parallelograms of forces.

Topic XII. Polygons

So far, we have considered figures that have used four or fewer lines. Suppose we now consider a quadrilateral to which a line is added connecting nonadjacent vertices. This line is called a diagonal. We see now that this diagonal line has divided our four-sided figure into one consisting of two triangles, and thus many of the things we have learned about triangles or parallel lines can be used in our study.

We should also consider closed figures of more than four sides and their diagonals. Once again we observe that all the previous information and facts about triangles are available to prove certain facts about these figures.

This now affords an opportunity to teach the sum of the angles, both interior and exterior, of any polygon as well as the number of degrees in each angle of a regular polygon.

Individual projects may be used to have pupils make models of various solids having different polygons as faces.

Topic XIII. Circles

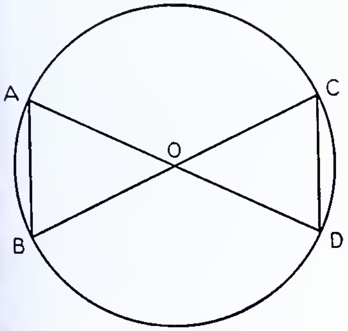
This topic can be initiated by reviewing and recalling to the pupils some of the facts which they have already encountered concerning circles. The teacher should be sure that the pupils have a clear concept of circle, radius, diameter, arc, and chord.

One of the most difficult things in presenting a lesson on circles is to insure that the pupils have a clear understanding of the meaning of "intercepted" and "subtended."

Emphasize practical applications such as pulley belts, gears, wheels, and other items which may be found in books on vocational mathematics.

Suggested approach to a proof concerning circles:

Don and Janice were studying their geometry lesson together one afternoon after school. Their class was studying about circles.



One of the problems required that a circle be drawn with two equal chords AB and CD, and that radii AO, BO, and CO, DO be drawn. It was then suggested they prove that triangle AOB is congruent to triangle COD.

"Oh, that's easy!" Don boasted. "Look! AO equals CO and BO equals DO because radii of the same circle are equal. Then, angle AOB equals angle DOC because they're vertical angles and triangle AOB is congruent to triangle COD because two sides and the included angle of one are equal respectively to two sides and the included angle of the other."

"Why, of course!" Janice agreed, and she began to write out her proof.

Indicate your position with regard to Don's proof by checking one of the following statements:

a. _____ I agree with Don's proof.

b. _____ I find fault with Don's proof.

If you have found fault with the proof, write your criticism.

Several concepts should be postulated such as: circles with the same or equal radii are equal, equal central angles intercept equal arcs, and equal arcs determine equal central angles.

From this point the teacher can carry the class quite readily into the proofs of as many of the standard textbook theorems concerning circles, chords, arcs, tangents, secants, and such facts concerning angle measurements as may be considered desirable and as time permits. Solid geometry aspects of the sphere and other figures should be taught concomitantly with the plane geometry in this and other units.

Topic XIV. Locus

This topic is adaptable to examples in everyday life. Such things as the path of the hub of an auto wheel, curbs and white lines along streets, swinging doors are examples.

The student should be impressed with the idea that the word *locus* comes from a Latin word meaning "place," that in mathematics locus refers to the path of a point moving according to certain conditions.

A starting point for this unit may be the circle, then the parallel lines, then angle bisectors, and finally line segment bisectors.

The teacher should note that this by no means exhausts the list of possible loci. Such items as ellipses, parabolas, and hyperbolas can be introduced and considered if the caliber of the class warrants such treatment.

Once the above material has been thoroughly mastered, the teacher should feel free to present the several theorems concerning angle bisections, medians, altitudes, and perpendicular bisectors of sides of triangles. This will present an opportunity to introduce such words as incenter, circumcenter, orthocenter, and centroid to the pupils.

Bring in examples of the many uses of loci—location of targets and forest fires, map reading, radio beams, weather maps. Pupils should be encouraged to construct models illustrating the loci theorems.

One of the best uses of films as an instructional aid is to demonstrate situations where motion is involved. Since locus is concerned with "the path of a moving point," here is an opportunity to utilize moving pictures.

In discussing various patterns of loci it is advisable to consider also situations in three dimensions. This brings out the relationship of a plane between two "walls" as well as the relationships commonly discussed in plane geometry.

Topic XV. Loci in Construction Problems

Here pupils will have an opportunity to use all the previously acquired knowledge about triangles, lines, circles, and loci. Situations will provide them with a challenge to test their ingenuity in using the various geometric facts and concepts.

Topic XVI. Ratio and Proportion

This topic is best introduced by starting with ratios, defining them, and recalling to the pupils all their previous experiences with ratios. References to algebra should be made. This will lead to proportions, which are a natural outgrowth of ratios.

Such operations on proportions as multiplication of means and extremes and its converse alternation, inversion, addition, subtraction, series of ratios, powers and the roots may be presented at this point with valuable results.

Scale drawings, maps, city planning, enlarging of photographs, weather maps, and models are a few of the types of applications which should be stressed. Here is an opportunity, as in the study of loci, to bring in resource people from the community to show how they use mathematical principles in their work.

Topic XVII. Correlation of Ratio and Proportion with Triangles and Parallel Lines

This can start with a study of the situation that a line joining the midpoints of two sides of a triangle divides these sides proportionally.

Topic XVIII. Similarity

This unit will come as a natural outgrowth of Topic XVII. It should be pointed out that this is a situation requiring only two facts about the triangles whereas congruence requires three facts. This is an excellent opportunity to point out the difference between congruency and similarity and thus clarify what makes triangles congruent or similar.

All the conditions which will make triangles similar can be introduced and the theorems demonstrated. These theorems will include: two sides proportional and the included angles equal; two angles equal and the included sides proportional; three sides proportional; the Pythagorean theorem.

The teacher should call attention to the fact that this development is parallel to the development on congruence.

Topic XIX. Indirect Measurement

The solving of problems involving indirect measurement provides an opportunity to utilize several mathematical approaches, namely:

A. By using congruent triangles (See Field Trip description on page 236.)

B. By using similar triangles

C. By using scale drawings

D. By using right triangles
   1. Pythagorean theorem
   2. Trigonometric ratios of sine, cosine, tangent

Here is an opportunity to introduce field instruments such as the angle mirror, the hypsometer, the sextant, and the transit. Pupils can make simple clinometers and determine inaccessible heights and distances around the school grounds.

Have the pupils report on the possibilities of the use of indirect measurement in measuring large areas, in aviation mathematics. (If possible, secure an instrument board used by aviators.)

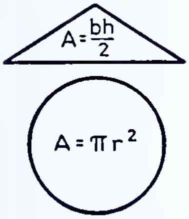
Explain how trigonometric ratios enable a pupil to find the height of a kite.

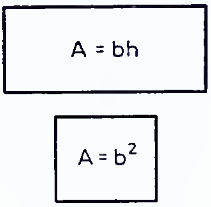
The trigonometry taught should be confined to right triangles only and should involve only sine, cosine, tangent, and the use of simple tables. However, here is an opportunity to do some articulation. The teacher should show how an entire branch of mathematics called *Trigonometry* is developed from the geometric concepts of similar triangles. Since this is a course to be followed by most pupils during part of the next year, it serves as a link to subsequent study in mathematics.

Topic XX. Equivalent Figures—Areas and Volumes

This topic is a continuation of the development of the ideas of congruence, similarity, and finally equivalence. This, of course, provides an opportunity to work into the area of geometric figures. These can be presented as areas of rectilinear figures. The difference between area and perimeter should receive special attention. A good sequence to follow might be rectangle, parallelogram, triangle, trapezoid. Proportional areas is a good second topic to show the dependence of areas on the various elements constituting the area, thus: two triangles having equal bases but unequal altitudes are to each other as their altitudes. Volume formulas should be developed along with the study of areas.

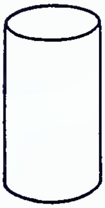
Plane Geometry





Experiment with various figures to see whether or not a circle includes a greater area than any other figure of equal perimeter.

Our Food Supply



Less tin is required to preserve the same quantity of food if round cans are used. Food in round cans can be stored economically.

Point out again that certain of the propositions in solid geometry are merely applications of theorems in plane geometry to plane figures lying in different planes in three-dimensional spaces. Thus the lateral surface of a pyramid is the sum of the areas of the triangles forming the faces; the lateral area of a prism is the sum of the areas of the parallelograms forming the faces. Similar points can be made about the lateral area of a right circular cylinder and a right circular cone.

Topic XXI. Inequalities

It certainly is as lifelike to find things unequal as it is to find equality. There are a number of geometric situations, axioms, and theorems which deal with inequalities. Logically they are geometric concepts and may well be considered under the appropriate topics, or the whole field of inequalities may be taught near the end of the course and serve indirectly as a summary.

The topics to be considered should include inequalities relative to

A. Axioms
B. Straight lines
C. Angles
D. Triangles
E. Circles—arcs and chords

Topic XXII. Inscribed and Circumscribed Regular Polygons

Here is a good place to introduce the theory of limits by the methods of exhaustion. This will also involve all the pupil's knowledge of circles and put it to good use. It provides an excellent opportunity for the pupils to do some meaningful constructions in finding angles of 18, 24, etc., degrees. This will lead to determination of circumferences, area of sector, segment, and arc.

ILLUSTRATIVE UNITS

UNIT I

GEOMETRY AS WE SEE IT ABOUT US

1. *Objectives*

   a. Major objective: An appreciation of geometrical forms in daily living

   b. Related objectives

      (1) To give construction immediate and definite purpose

      (2) To develop an awareness of the many uses of geometrical forms in art, advertising, flags, textiles, designing, architecture, insignia, etc.

   c. Indirect and incidental objective: Increased awareness of some of the optical illusions created by geometrical figures, or what may be done with geometry to twist the facts for certain advantages. (Relate this to distorted reasoning in all areas of living.)

   d. *Prior Masteries:* Using only compass and straight edge, the pupil should be able to execute all of the fundamental constructions that go with a course in plane geometry.

2. *Suggested Pupil Activities*

   a. Select geometrical forms found in advertising, insignia, flags, transportation, art, architecture, industry, sports, trademarks, church windows, stamps, bridges, nature, etc. (Illustrations may be prepared by the teacher showing a few outstanding items, then passed about the class and later posted on the bulletin board of the mathematics room where pupils can see clearly the manner in which the work is to be carried out.) See pages 212, 263, and 303.

      (1) Group these under some one heading such as "Geometry in Tire Treads" or " Geometry in Trademarks or Trade Names." Other suggestive headings are: "Geometry in Flags," "Geometry in Cartoons," "Geometry in Advertising," "Geometry in Nature," "Geometry in Aircraft Marking," "Geometry in State Flags," "Geometry in Gasoline Signs."

      (2) Mount the clippings or findings on a sheet of mounting paper. Use upper third of sheet for this purpose.

      (3) On the same sheet, make the basic construction used in the clipping (see examples). If one of the forms selected is the Texaco star, the pupil should note that the basic construction is the pentagon.

b. Try to get together a variety of geometrical forms, as the square, rectangle, circle, pentagon, hexagon, triangle, ellipse, or perhaps a combination of forms. Make as large a collection as possible.

c. Design a cover for your booklet in keeping with your subject.

d. Discuss design in nature; symmetry; why some things are pleasing to the eye. Explain the geometric basis for some of the designs. The pupils should make the constructions needed to carry out their work.

e. Discuss good taste in design. Show why manufacturers often turn to geometry for designs for their trademarks when they have a new product to place before the public. The trademark of the National Biscuit Company is said to be worth 2 million dollars to its owner. Point out some advantages in the use of geometric forms for airplane insignia, railroad identifications, gasoline trademarks, etc.

3. *Evaluation*

Final Test—Pass the booklets about the class and note the interest manifested by the students as they view what each of the others has done.

4. *Materials and Apparatus*

Compasses, rulers, paste, scissors, light-weight mechanical drawing paper, light-weight cardboard for covers, a variety of magazines carrying commercial advertisements, and many other sources for discovering new ideas.

5. *Outcomes*

After each pupil has had a chance to see the work of the others, he should be thoroughly convinced that geometry is in wide use about him. Some see it in nature; others see it as planned by the hand of man. Another outcome that must not be overlooked is that work on the unit often indicates the trend of the pupil's interest. In 1941 "Geometry in Aviation Insignia" was most excellently prepared in one school by a boy who in 1944 was a navigator for the U. S. Air Forces over Europe. "Geometry in the Military" was a theme selected by a boy interested in the N.R.O.T.C. "Geometry in the Home" was presented by a boy interested in architecture. Others will show similar interests.

UNIT II

STREAM MEASURING FIELD TRIP IN GEOMETRY

1. *Overview*

It has been contended by many teachers that plane geometry is too theoretical a subject to use field trips to any extent, but such field trips are valuable if planned carefully. This is an illustrative unit which has been worked out successfully. First, pupils must clearly understand the theorem upon which this unit is based; then the class discusses the problem thoroughly and makes complete plans as to what each pupil is to do. Every pupil has a task. A drawing of the task is made on the board and every why and wherefore is discussed.

The problem is to find the distance across a stream without crossing that stream. The theorem upon which it is based is: "If two angles and the included side of one triangle are equal respectively to two angles and the included side of another, the triangles are congruent." The materials or equipment needed are one ball of string and a hatchet, a tape measure or yardstick for finding the distance in feet and inches and for checking the distance. Later it is interesting to measure the actual distance across the stream to find the percentage of error. It helps to make the unit more significant when the pupils can see how accurate they have been.

The following plans show how to set up the unit and how to carry it out. This is a pupil-participation program: they do all the planning and the work; the teacher merely acts as an adviser.[1]

2. *Preliminary Theory*

Check over the following theorem (which may well be postulated) for its meaning, "If two angles and the included side of one triangle are equal respectively to two angles and the included side of another, the triangles are congruent."

3. *The Problem Analysis*

If we can prove that triangle ABF is congruent to triangle LDF, then DL will be equal to AB and measuring DL will give us the distance across the stream.

Given: AB is perpendicular to BD, DL is perpendicular to BD, F is the midpoint of BD

Prove: That triangle ABF is congruent to triangle LDF

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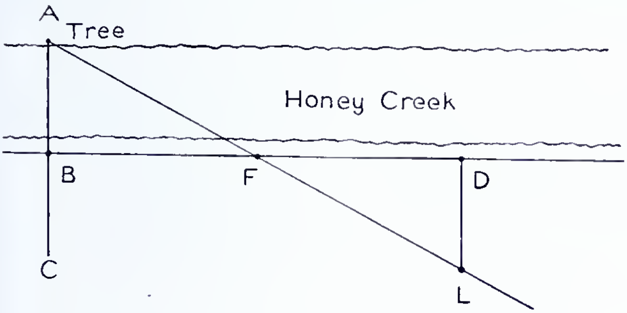
[1] *Field Work in Mathematics* by Shuster and Bedford is a very helpful source book for teachers interested in applied geometry.

Analysis: If I can show that the two angles and the included side of angle ABF are equal respectively to two angles and included side of angle LDF, then they are congruent.

Since AB and DL are perpendicular to BD, then the angles at B and D are equal.

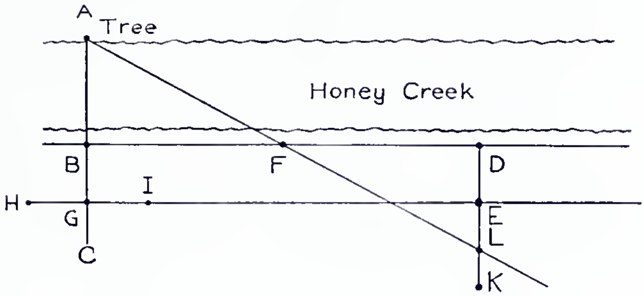
Since F bisects BD, then BF equals DF. With two angles and the included side equal in both triangles, then they are congruent. Since they are congruent, DL is equal to AB.

4. *Planning the Work*



a. Equipment*
   (1) String
   (2) Hatchet
   (3) Tape Measure

b. Cut eight stakes*



c. Construct line AC. Line stake at C with one at B and tree across the stream at A.*

d. Construct line HI perpendicular to BC and mark H and I at equal distances from B and C. Extend line HI by lining up E and H and I.*

e. Construct BD parallel to GE. Construct line DE to be perpendicular to HE in the same manner as item d above, checking by diagonals BE to be equal to DG.*

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* Indicates where a pupil or two should be assigned specific responsibility.

f. Bisect line *BD* at *F*. Double *B* back to *D* until *BF* equals *DF*.*

g. Construct line *DK*. Line *K* with *D* and *E*.*

h. Find point *L*. Move along line *DK* until *L* lines up with stake at *F* and tree across stream at *A*.*

i. Measure line *DL*. Use tape measure.*

j. Measure *AB*. Pupil and instructor cross stream and instructor fastens a stone to end of line and then in manner of sling throws it across to another pupil who takes it to tree and then the instructor draws it taut and marks line at *B*. String is then drawn across and measured for actual distance.

5. *Results*

Calculated distance by measuring *DL*	105′2″
Actual distance by measuring *AB*	108′9″
Difference	3′7″

6. *Instructional Aids*

Geometry is particularly adapted to the use of instructional aids. Most of the concepts and relationships can be readily visualized when reinforced by examples from life or by the use of special devices. Teachers should be thoroughly familiar with the *Eighteenth Yearbook,* with *Field Mathematics* by Shuster and Bedford, and with the laboratory concept as described in this chapter. Much of the learning in geometry comes through pupil construction of models, designs, and devices. Throughout the course the teacher should use colored chalk to indicate important relationships in figures drawn on the board.

In addition there are numerous commercial aids (in addition to films) which the teacher should consider. Some of these are:

The Perspectograph—a templet devised to facilitate the drawing of three dimensional figures. (Margaret Joseph, Milwaukee 2, Wisconsin.)

Schacht's Dynamic Geometry Instruments. (Welch Scientific Company.)

Speed-Up Geometry Ruler. (Speed-Up Geometry Ruler Co., Inc., 2260 Elsinor Avenue, Baltimore 16, Maryland.)

SPECIALIZED MATHEMATICS

GRADE TWELVE

THEME: *Mathematics for the Consumer*

ATTENTION TO BASIC MATHEMATICS

The first semester of this year's work should be devoted to the study of trigonometry. In addition to the new trigonometric knowledge which this course offers, it should also serve as a valuable and practical educational experience by way of reviewing and unifying the important topics of arithmetic, algebra, and geometry. The teacher should be alert to the many practical applications that can be found to stimulate the pupils' study of mathematics.

The second semester's work might be called "Advanced Mathematics." Several possibilities may be pursued, depending on the plans which the pupils have for the following year. It is suggested that a semester's work be formulated from the areas suggested below. The actual selection should be determined by the special mathematical needs of the pupils concerned. The second semester's work in the twelfth grade should be introductory and should not be thought of as replacing the college courses; e.g., no attempt should be made to cover the college analytic geometry course. Rather than a smattering of all the suggested topics, it would be *preferable to select the one or two topics of the greatest value* to the group concerned and to allow time for a thorough study of these topics.

Basic Mathematics

It should be emphasized that the essence of this guide is for all pupils to have training in the common learnings of mathematics. In the Specialized Mathematics for the twelfth grade, teachers should give attention to the mathematics for daily living by:

1. Studying and utilizing the outline for Basic Mathematics suggested for the twelfth year
2. Emphasizing the practical application of trigonometry, algebra, and statistics
3. Stressing particularly the understandings in those topics marked *Basic* in the following outlines. These topics have comparable areas in Basic Mathematics. The essential difference is that in the Specialized Mathematics the concepts and the techniques will be extended further than in the similar topics as presented in the Common Learnings Program.

Consumer Education

To insure some training in the consumer learning of mathematics, the equivalent of one period per week should be devoted to consumer mathematics. It is suggested that this part of the program be followed throughout the year rather than be concentrated in a short period. Teachers will find helpful material and suggestions in the resource unit, Family Financial Security, which is included in the Basic Mathematics outline for Grade 12.

TRIGONOMETRY (one semester)

I. *Introduction*

A good course in trigonometry reveals the relationships of arithmetic, algebra, and geometry to the area of mathematics known as trigonometry and affords excellent review opportunities in these branches. From the standpoint of new material, it serves as a prerequisite for advanced mathematics, especially analytic geometry and calculus. As such it serves to meet the special needs of some pupils while presenting an opportunity to understand the significance of mathematics in the development of society.

Whether the pupil studies trigonometry for cultural, personal, or preprofessional reasons, he should develop basic knowledge of the significance and the meaning of trigonometry. The teacher should stress a developmental approach that will reveal the why's and wherefore's, thus emphasizing the understanding of trigonometry. Rather than try to cover a multiplicity of topics related to trigonometry, it is the intent of this guide to build a firm foundation which will serve as a base upon which college courses and specialized courses of industrial concerns may be built.

The mathematical competence which trigonometry aims to establish centers around two areas—the study of triangles and the development of the analytic properties and relationships of the trigonometric functions. Neither of these two areas should be stressed to the exclusion of the other. The pupil will have developed some understanding of the solution of triangles principally through some consideration of numerical trigonometry in earlier courses in mathematics. To build upon this background, the outline suggests topics related to the solution of right triangles.

The teacher should be concerned with applications of trigonometry to life problems and, wherever possible, to school experiences. Most of the elementary physical applications occur in the solution of triangles and these applications should be introduced into the course at frequent and appropriate intervals. Most good texts contain a variety of such problems, and a role of the teacher is to have at his disposal a number of such sources so that he may select those problems which seem best related to the interest and to the needs of the pupils in each class.

II. *Objectives*

The following goals in terms of pupil development constitute the objectives of the course in trigonometry. They may be regarded as guides to teachers in directing pupil growth.

The study of trigonometry should assist pupils

1. To expand their appreciation of the contributions of mathematics to society with emphasis upon the power of indirect measurement
2. To increase their understanding of mathematical terminology and symbolism, including the understanding of graphs of social and commercial data
3. To gain efficiency and understanding in the analysis and the solution of life problems involving computation
4. To understand the ways in which and for what purposes certain trigonometric formulas are developed
5. To develop a comprehension of trigonometry as a related part of the larger field of mathematics
6. To grow in the ability to use understandingly mathematical tables and other aids in computation
7. To understand the meaning and the uses of trigonometry
8. To develop the ability to use trigonometric functions properly.

III. *Outline of Activities by Topics*

TOPIC 1. The Meaning of Trigonometry—*Basic*
TOPIC 2. Functions of General Angles
TOPIC 3. Fundamental Trigonometric Identities
TOPIC 4. Radians
TOPIC 5. Graphs of Functions
TOPIC 6. Solution of Right Triangles—*Basic*. Approximate Numbers—*Basic*
TOPIC 7. Trigonometric Formulas
TOPIC 8. Solution of Oblique Triangles
TOPIC 9. Logarithms
TOPIC 10. Solution of Triangles by Logarithms
TOPIC 11. Optional Topics

The outline which follows is divided into two main parts: The first ten sections, which form the core of the trigonometric course, and the eleventh section which contains suggested optional topics for more capable pupils or for mathematics club activities.

CONTENT	APPLICATIONS	SUGGESTIONS
1. The meaning of trigonometry. The measurement of angles. Functions of acute angles — especially those of 30, 45, and 60 degrees. Solution of right triangles as a limited application *Basic*	Height of a building Width of a river Vector problems Scale drawing	Early use of slide rule Relate to algebra and geometry (Similar triangles)
2. Functions of general angles, special angles Quadrants	Components	
3. Fundamental trigonometric identities		Pupils should familiarize themselves with ways of changing standard forms into relationships such as $\sin^2 A - 1 - \cos^2 A$
4. Radians Length of arc Area of sectors, segments	Change RPM to radians per sec.	Interchange radians and degrees with and without use of tables. Omit problems involving complicated arithmetic such as "change 137° 248′ 43″ into radians without the use of tables."
5. Graphs of sin A cos A tan A	Simple harmonic motion Vibrating spring Alternating current	In graphing use both radians and degrees to show the proper relationship between the limits on the vertical and on the horizontal axis.
6. Solution of triangles by natural functions Approximate numbers *Basic*		Point out that most trigonometric data are secured through measurement and hence are approximate.

CONTENT	APPLICATIONS	SUGGESTIONS
7. Trigonometric formulas Sum and difference of two angles Double-angle Half-angle Product	Harmonics	Stress the generality of these formulas, e.g., $\sin 2A = 2 \sin A \cos A$ and $\sin 4A = 2 \sin 2A \cos 2A$ are two forms of same formula
8. Oblique triangles Sine law Cosine law	Mensuration of triangles	Use of instruments (See *Field Work in Mathematics* by Shuster and Bedford)
9. Logarithms Computation — products, quotients, powers, roots Various bases Logarithm formulas	Slide rule *(Basic)* Business application $A=P\ (1 + r)^n$	The pupil should know the meaning of logarithm, as well as rules of manipulation
10. Solution of triangles by logarithms Right triangles Oblique triangles with two angles and a side given		

11. Optional topics—to be considered when, where, and if the teacher feels the group or some members can profit from them

Inverse functions

Trigonometric equations

Other cases of oblique triangles solved by logarithms

Demoivre's theorem

Trigonometric equations involving multiple angles

Graphs of cot x, sec x, csc x.

Mils

Spherical trigonometry

Area of a triangle

Polar coordinates

ADVANCED MATHEMATICS (one semester)

I. *Outline of Activities by Topics*

(Note that this work is planned for one semester and as such all five areas should not be studied—only the ones meeting the pupil needs of that year.)

Selections from

1. Review Mathematics
2. Advanced Algebra
3. Statistics—*Basic*
4. Demonstrative Solid Geometry
5. Analytic Geometry

1. An extensive review of all high school mathematics with emphasis placed upon the pulling together of the diverse subject elements so that the pupil can see mathematics as an integrated body of subject matter and processes and not as a body of unrelated divisions of arithmetic, algebra, and trigonometry.

2. Advanced algebra dealing with the extension and clarification of some of the more difficult concepts of Algebra II, especially the binomial theorem and progressions, complex numbers, basic work in the theory of equations, permutations, combinations, simple work in probability, and elementary work in determinants.

3. Statistics dealing with problems concerned with gathering and displaying data, interpreting graphical representations, mean, median, mode, and some measures of dispersion—*Basic*.

4. A four weeks' course in demonstrative solid geometry to supplement the concepts presented in Geometry, Grade Eleven.

5. Analytic geometry dealing with the relationship between algebra and geometry, plotting of the curves and discussion of the equations of straight lines, circles, and simple cases of the conics. These topics could be studied as parts of the advanced algebra.

It is difficult to outline a satisfactory program for the mathematical genius who may appear in this class. It is hoped that the teacher will spot such individuals and will outline a program suited to their capacities.

Of the above suggested topics, a detailed outline is given below only in the case of item 3—Statistics. There are numerous satisfactory textbooks for the other topics.

Mathematics and the Curriculum

THOSE responsible for the development of curriculum improvement programs involving mathematics are more and more interested in the bases upon which decisions are made. In recent efforts more attention has therefore been given to the selection, organization, and teaching of learning experiences. This is in contrast with curriculum improvement of the past which devoted its primary emphasis to listing the special content to be covered. Curriculum development in the field of mathematics continues to be concerned with vertical continuity. In addition, the relationship of the mathematics program to the rest of the curriculum is given greater consideration.

Mathematics content, per se, has changed little. Changes have been made, however, in the uses to which mathematics is put, in the ways pupils study and learn mathematics, in the mathematical procedure and materials used, and in the emphasis on developing understanding and thinking in mathematics.

Teachers, research and curriculum workers, and psychologists have directed attention to the need for making teaching and learning more meaningful and significant to pupils, for tying together pupils' experiences, for confronting pupils with content and situations which are appropriate to their experience, their maturation, and their capacity. This has sometimes been described as developmental teaching—teaching contrasted with verbalistic learning having little meaning or significance to the learner.

A "good" curriculum in mathematics stresses *meaning*—that quality of learning which helps the pupil to make sense out of what is being learned. It shows "relationships" among ideas and processes. Inventiveness is encouraged. And a premium is placed on *thinking*.

—CURRICULUM AND MATERIALS
Board of Education, City of New York.

Elementary Statistics

A. OBJECTIVES

The objectives of a course in Elementary Statistics are

1. To develop an understanding of and a familiarity with the elementary and commonly used statistical vocabulary
2. To develop the ability to collect, organize, and interpret simple statistical data, especially as found in everyday social problems
3. To develop the ability to understand, interpret, and evaluate statistical comments and graphs found in newspapers and periodicals
4. To give an orientation and overview of the possibilities and opportunities in the field of statistics
5. To develop increased competence in arithmetic

This course should not be limited to technical students, and the standards should be set so that pupils with a good working knowledge of arithmetic can profit from the course. The teacher presenting this to a Basic IV group should be prepared to teach the small amount of algebra which is needed.

The most difficult and important part of statistics is an accurate appraisal of the results. It is especially important that the pupil understand the hypotheses upon which his conclusions are based and that he realize that one of the primary purposes of statistics is to form conclusions about the characteristics of a large group on the basis of the study of a properly selected sample. Attention should be paid to developing an understanding of the part that statistics play in modern life.

B. CONTENTS OF ELEMENTARY STATISTICS

1. *Tabular Data*

   Methods of collecting and organizing data. Pupils should study local phenomena and make tally sheets, tables, and graphs of the data in such topics as height of high school pupils, school enrollment, living costs, wages, etc.

2. *Graphs*

   Bar graph, histogram, pictogram, frequency polygon, frequency curve, and ogive. Pupils should be encouraged to interpret and criticize graphs which appear in current newspapers and periodicals.

3. *Averages or Measures of Central Tendency*
   Various types of averages such as arithmetic mean, median, mode, and (as optional topics for the more capable pupils)

THE OUTDOORS BECOMES THE CLASSROOM

geometric mean, harmonic mean. It should be pointed out that the first two of these are widely used. The applications of these should be discussed and the effect of extreme values on the arithmetic mean and median should be examined.

4. *Quartiles, Percentiles*

The same methods used for finding the median can be used here. Since most graduating classes are divided into percentiles, pupils already have some conception of these terms.

5. *Optional Topics*

Short methods for computing the arithmetic mean, quartile deviation, standard deviation, correlation. If the short methods are included, emphasis could be given to the checking of the results in order that the pupil may develop confidence in his ability to carry out an arithmetic process correctly.

C. REFERENCES FOR TEACHING STATISTICS IN HIGH SCHOOL

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Hausle, E. C., Benjamin Braverman, Harry Eisner, Max Peters. *Mathematics You Need,* New York, D. Van Nostrand Company, Inc., 1949. Chapter 12—"How Can You Get Facts from Figures?"

Pollard, H. S., "Liars and Statisticians." *School Science and Mathematics,* 1945, 45:569-576.

Potter, A. A., "When and How Do Figures Lie? Mathematical Theory of Measurement and Price as Basis for Moral Judgment of Social Action." *Commercial and Financial Chronicle,* Vol. 162, July 5, 1945, p. 69 +.

Scates, D., "Statistics—the Mathematics for Social Problems." *Mathematics Teacher,* Vol. 36 (February, 1943), pp. 68-78.

Schorling, Clark, Lankford, *Statistics.* World Book Company, 1943.

Schorling, R., et al., "Statistics for High School Students." *Secondary Education,* 1944, 11:5.

Smith, O. S., et al., "We Recommend the Teaching of Statistics in High School." *Mathematics Teacher,* 1946, 39:182-183.

Walker, Helen M., "Mathematics and Statistics." *Sixth Yearbook* of the National Council of Teachers of Mathematics, pp. 111-135.

Walter, R., "Statistical Training as a Part of General Mathematical Education." *New Jersey Mathematics Teacher,* 1949, 6:5-8.

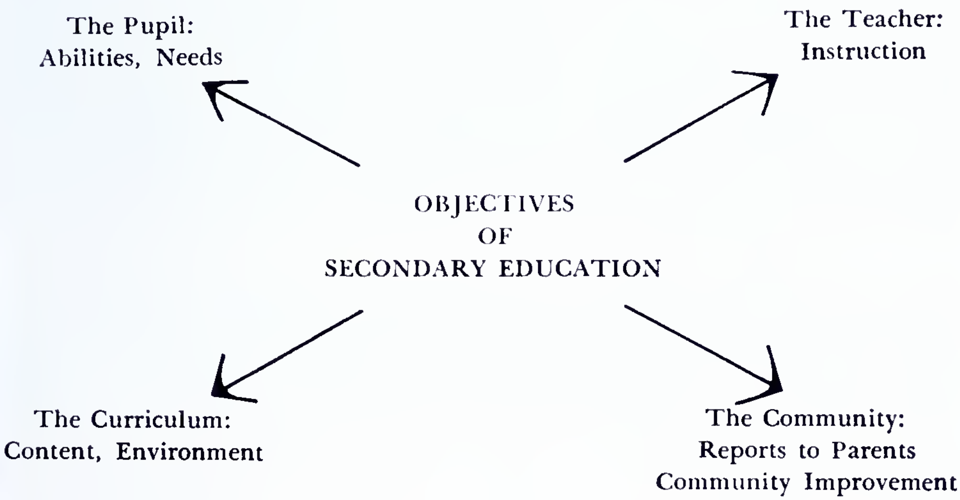
CHAPTER IV

EVALUATION IN MATHEMATICS

A continuous evaluation program should be a part of an instructional improvement organization. "Every attempt should be made to determine the efficiency of the organization; the emotional climate in the school; the attitude of teachers, pupils, and parents toward the changes [in the curriculum]; and the feeling of teachers concerning opportunities for participation. In fact, the staff should take ample time to plan a detailed appraisal program."[1]

The following diagram indicates the areas of concern in evaluation.

EVALUATION OF PUPIL PROGRESS



PURPOSES OF EVALUATION

The evaluation program should be a cooperative enterprise of all those who affect and all those who are affected by the school program. In its larger aspects, evaluation is an integral part of curriculum development and helps the educator to discover: Are we moving in the right direction? How far have we gone in that direction? In its narrower (but equally important) interpretation, evaluation is a concern of the individual teacher as to how the classroom activity under his direction is affecting the development of each individual pupil.

While evaluation is occasionally used synonymously with "test to get a mark," teachers of mathematics are increasingly using it

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[1] H. R. Douglass, ed., *Education for Life Adjustment.* New York, Ronald Press, 1950, page 297.

1. For improvement of instruction—appraisal of the teacher himself and of his instructional methods
2. For pupil self-evaluation—increasing the ability to be self-reliant
3. As a basis for guidance—securing information of pupil background
4. As a device to discover how well individuals are learning to study in mathematics
5. For measuring "intangibles"—understandings, ideals, attitudes, behaviors
6. As a basis for reporting progress to parents
7. For maintaining certain standards
8. For accrediting the school
9. For accumulating materials for research

Essentially here is an attempt to measure pupil progress toward stated objectives. To do this, teachers and administrators must collect, analyze, and interpret information about pupils. This will help both the pupils and their teachers to determine whether or not the goals of the mathematics program are being attained. In short, are the pupils learning what they need to learn? If the answer is in the negative, the objectives should be re-examined. Are they appropriate? Are they attainable? The weakness may also lie in the curriculum content, in the instruction, or in the faulty interpretation of the learner's abilities and needs.

The desired outcomes should stem from the concept of education which may be defined as: A process which seeks to change for the better the way in which human beings behave—think and act and feel. In Chapter I the objectives of the mathematics program were proposed. These need to be appraised as to their contribution to the purposes of secondary education.

OUTCOMES WHICH ARE TO BE EXPECTED

The mathematics program should be appraised for the following outcomes:

1. Skills, understandings, and attitudes which will make the learner an intelligent and productive participant in the economic world to which he must make social adjustments
2. Physical fitness, mental and moral well-being as a consumer of the world's goods and services
3. Appreciation of the importance of family, social, and other group institutions for a healthy national welfare
4. Understandings of one's rights, duties, and obligations as a member of a democratic and free way of life and government

5. Use of scientific methods and use of reasoning in evaluating all the controversial issues which confront the pupil as a member of a productive and progressive community
6. An appreciation of the cultural, and aesthetic aspects of the civilization of which the pupil is a part and to which he can make a definite contribution by way of further refinements
7. A wide range of significant interests and effective methods of thinking
8. Useful work habits and effective study techniques

These outcomes can be evaluated in many ways, but one of the most effective devices is to set up situations involving the pupil in such a way that his responses indicate the amount and direction in which his behavior patterns are being influenced and trained.

In order to accomplish this, whole new blocks of ideas will have to be developed and formulated about the existing methods of testing and evaluating. The teacher will have to become more and more aware of the limitations of the existing evaluating instruments and be led to realize that the only effective device is one which involves the pupil in such a way as to reveal through his responses the way in which he is reacting to situations.

This idea of testing is of particular interest to the teacher of mathematics, for he should not desire the pupil to acquire a collection of isolated bits of subject matter but rather some broad concepts and techniques which, when skillfully and wisely applied to the situations confronting him, will assist him in making a wise and useful adjustment to life.

Following are some sample evaluating devices which may serve as models for the development of new evaluating techniques. When the techniques and findings of many teachers are collected, analyzed, and organized, a helpful pattern can be presented to indicate the way to more effective testing and evaluation.

METHODS FOR EVALUATING OUTCOMES

I. Standardized Tests
    A. Prognostic
    B. Diagnostic
    C. Aptitude
    D. Achievement (both general and mathematical)

II. Evaluative Criteria[1]

While this is generally used for an all-school appraisal, teachers of mathematics and mathematics departments should study Section D-11 on Mathematics in the Secondary School. Self-evaluation, using the *Criteria,* is vital. By permission of the Cooperative Study of Secondary School Standards, Section D-11 is reproduced:

STATEMENT OF GUIDING PRINCIPLES

The mathematics curriculum consists of those courses, activities, and units of instruction which are designed to contribute to the common and specialized mathematics needs of secondary school pupils. The curriculum places particular stress on developing accuracy and facility in the use of computational processes in practical situations. Provisions are made for pupils to develop skills and abilities in the mathematics areas of geometry, graphic representation and interpretation, elementary analysis, and scientific thinking.

The curriculum in the advanced grade levels provides for a sequential presentation of general and special mathematics contributing to present and probable future educational, vocational, and cultural mathematics needs of individual pupils. The activities are selected and organized with due regard for such psychological principles of learning as those concerning readiness, motivation, rates of learning, and degrees of mastery. Constant attention is given in the learning process to the fundamentals of mathematics, while at the same time significant applications are made within the learner's potential range of understanding and interest.

ORGANIZATION

CHECK LIST

( ) 1. Sequential courses or activities in mathematics are required of all pupils in grades 7, 8, and 9.

( ) 2. Pupils who show competency in and need for mathematics are encouraged to take additional mathematics beyond the ninth grade.

( ) 3. Advanced general mathematics courses or activities are available in grades 10, 11 or 12.

( ) 4. Sequential mathematics courses or activities are available in grades 9 through 12 to meet specialized mathematics needs of individual pupils. (Describe under "Comments." Indicate if separate programs are available for groups of pupils having particular needs—college preparatory, business, technical, etc.)

( ) 5. Pupils are assisted through counseling in electing mathematics courses.

( ) 6. Remedial instruction in mathematics is available.

( ) 7. Provision is made for reassignment of pupils to other mathematics courses when desirable.

( ) 8. Each course is generally organized into a few comprehensive units built around key concepts and fundamental principles.

( ) 9. Other.

( ) 10. Other.

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[1] See item 7 of the bibliography on page 269.

EVALUATIONS

( ) *a. To what extent are general mathematics courses provided for all pupils?*

( ) *b. To what extent are mathematics courses available for pupils with specialized mathematics needs?*

( ) *c. Do the time allotments for mathematics activities meet mathematics instruction needs satisfactorily?*

( ) *d. To what extent do the enrollments in mathematics show that the needs of all pupils for instruction in mathematics are being met?*

NATURE OF OFFERINGS

CHECK LIST

Factors used in the selection of curricular activities include

( ) 1. Pupil readiness to utilize the mathematical processes involved.

( ) 2. Contributions of the processes to an understanding of mathematics in problems of living.

( ) 3. The need for a logical sequence in developing knowledges and skills.

( ) 4. Probable future needs and aims of pupils.

( ) 5. Other.

The mathematics curriculum in the area of number and computation includes such experiences as

( ) 6. Understanding and mastering the fundamental number and computational operations.

( ) 7. Making practical applications of the fundamental number and computational operations.

( ) 8. Understanding and appreciating numbers and the techniques for analyzing computational situations.

( ) 9. Estimating and verifying answers.

( ) 10. Other.

The mathematics curriculum in the area of geometric form and space perception includes such experiences as

( ) 11. Developing understanding of a variety of geometrical figures.

( ) 12. Interpreting elementary geometrical relations.

( ) 13. Testing geometrical principles and relations.

( ) 14. Making practical applications of geometrical principles and relationships.

( ) 15. Other.

The mathematics curriculum in the area of graphic representation includes such experiences as

( ) 16. Developing graphical representations from a variety of data.

( ) 17. Reading and interpreting graphic materials.

( ) 18. Applying graphical representations to daily-life situations.

( ) 19. Relating graphical representations to elementary statistics.

( ) 20. Developing an understanding of the advantages, disadvantages, and dangers present in the use of graphs.

( ) 21. Other.

The mathematics curriculum in the area of elementary analysis includes such experiences as

( ) 22. Understanding the terminology of elementary analysis (e.g., negative number, exponent, cosine, average).

( ) 23. Applying the fundamental principles of algebra and elementary analysis in basic techniques (e.g., solving equations, solving triangles).

( ) 24. Making meaningful applications of principles of elementary analysis in mathematical situations.

( ) 25. Understanding and appreciating the implications of elementary concepts of functional mathematics relationships.

( ) 26. Other.

The mathematics curriculum in the area of logical thinking includes such experiences as

( ) 27. Using correctly such terms as assumption, proposition, converse, and conclusion in meaningful situations.

( ) 28. Understanding the basic principles and assumptions on which the structure of mathematics rests (e.g., manipulative techniques, grouping, distribution).

( ) 29. Recognizing the differences between logical and illogical quantitative thinking.

( ) 30. Applying principles of coherence and logic in meaningful mathematical situations.

( ) 31. Emphasizing the basic principles and techniques involved in establishing a valid proof or argument.

( ) 32. Other.

The mathematics curriculum in the area of relational thinking includes such experiences as

( ) 33. Understanding the relationship between variables.

( ) 34. Determining and expressing the relationship between variables.

( ) 35. Constructing simple tables, such as frequency tables, from raw data.

( ) 36. Constructing tables from formulas.

( ) 37. Other.

The mathematics curriculum in the area of symbolic representation and thinking includes such experiences as

( ) 38. Translating quantitative statements into symbolic form and vice versa.

( ) 39. Understanding the significance of the correct use of symbolic techniques.

( ) 40. Learning how to interpret the results secured through the use of mathematical processes.

( ) 41. Other.

EVALUATIONS

( ) *a. How adequate is the variety of offerings in mathematics to meet needs of all pupils?*

( ) *b. How adequate is the content of offerings to develop mathematical knowledges and skills needed by all pupils?*

( ) *c. How adequate is the content of the offerings to develop mathematical understandings and appreciations needed by all pupils?*

( ) *d. To what extent do the offerings provide for the application of mathematics in situations which are meaningful to pupils?*

( ) *e. How adequate is the content of offerings to develop mathematical* **knowledges** *and* **skills** *needed by pupils with specialized (including vocational and technical) needs and interests?*

( ) *f. How adequate is the content of offerings to develop mathematical* **understandings** *and* **appreciations** *by pupils with specialized (including vocational and technical) needs and interests?*

PHYSICAL FACILITIES

CHECK LIST

( ) 1. Drawing tables or drawing surfaces are available.
( ) 2. Shelves and bookcases are provided.
( ) 3. A showcase or display area is provided.
( ) 4. Facilities are available for the use of films and other audio-visual aids.
( ) 5. A large graph board or cross-sectional blackboard is provided.
( ) 6. A spherical blackboard is provided.

The following instruments and supplies are available for pupil use:

( ) 7. Blackboard instruments.
( ) 8. Compasses.
( ) 9. Slide rules.
( ) 10. Protractors.
( ) 11. Parallel rules.
( ) 12. A pantograph.
( ) 13. A vernier and vernier caliper.
( ) 14. A micrometer.
( ) 15. Surveying instruments suitable for school use.
( ) 16. A sextant suitable for school use.
( ) 17. Calculating machines.
( ) 18. Such general supplies as colored chalk, Bristol board, pasteboard, and tracing materials.
( ) 19. Various types of graph paper.
( ) 20. Models such as prisms, pyramids, cones, spheres, and polyhedrons.
( ) 21. Models of mathematical instruments, such as an abacus, sundial, and navigational instruments.
( ) 22. Other.
( ) 23. Other.

EVALUATIONS

( ) *a. How adequate are the space provisions for existing class sizes?*
( ) *b. How adequate is the equipment to meet enrollment and curricular needs?*
( ) *c. How adequate are the supplies to meet enrollment and curricular needs?*
( ) *d. How adequate are the storage facilities for equipment and supplies?*
( ) *e. How effectively do pupils use the available equipment and supplies?*

DIRECTION OF LEARNING

A. Instructional Staff

CHECK LIST

All members of the mathematics staff

( ) 1. Have had background preparation in such subjects as history of mathematics, theory of equations, mathematics of finance, statistics, and determinants.
( ) 2. Have had preparation in such mathematics subjects as algebra, trigonometry, calculus, Euclidean and non-Euclidean geometry, analytic geometry, and solid analytics.
( ) 3. Have had experience in advanced algebra emphasizing critical analysis as well as manipulation.
( ) 4. Have taken courses which include the study of the contributions of mathematics to general education.
( ) 5. Have had background experience in practical fields of business or industry where mathematics is used.
( ) 6. Have had preparation in methods of teaching mathematics.
( ) 7. Have had preparation in educational measurements or educational statistics sufficient to prepare them to assist other members of the staff with measurement problems.

( ) 8. Maintain acquaintance with recent developments in the teaching of mathematics.

( ) 9. Have had background experience and preparation which placed emphasis on the "why" as well as the "how" in mathematics.

( ) 10. Others.

( ) 11. Others.

EVALUATIONS

( ) *a. How adequate is the preparation of the staff in mathematics?*

( ) *b. How adequate is the preparation of the staff in methods of teaching mathematics?*

B. Instructional Activities

CHECK LIST

( ) 1. Instruction in mathematics contributes to the school's objectives.

( ) 2. Instruction is directed toward clearly formulated, comprehensive or long-range objectives in mathematics.

( ) 3. Specific instructional activities contribute to the comprehensive objectives of the mathematics program.

( ) 4. There is evidence of careful planning and preparation for the instructional activities.

( ) 5. Instructional activities are readily adapted to new or changing conditions.

( ) 6. Drill activities are meaningful to pupils.

( ) 7. Flexible or differentiated assignments are used to make provision for needs of individual pupils.

( ) 8. Resources of the community and environment are used whenever possible to make practical applications of mathematics.

( ) 9. The instructional activities within the classroom are adapted to individual aptitudes and abilities by providing experiences which vary in difficulty.

( ) 10. Laboratory and other investigational techniques are used in conducting the activities.

( ) 11. The mathematics activities include emphasis upon cultural and scientific uses.

( ) 12. The mathematics activities are integrated with other curricular areas.

( ) 13. Use is made of models, charts, and other instructional aids.

( ) 14. Movies and slides are used in the instructional activities.

( ) 15. Appropriate provision is made for both individual and group instruction.

( ) 16. Instructional activities are integrated with mathematics club activities.

( ) 17. Other.

( ) 18. Other.

EVALUATIONS

( ) *a. How adequate is the planning and preparation for the instructional activities?*

( ) *b. How adequately are the instructional activities adapted to the needs of individual pupils?*

( ) *c. How well are the assignments adapted to needs of individual pupils?*

( ) *d. To what degree are laboratory techniques used in the classroom activities?*

( ) *e. How adequately do the activities provide for practical applications of mathematics?*

C. Instructional Materials

CHECK LIST

( ) 1. A variety of modern mathematics textbooks is provided.
( ) 2. A variety of general and technical reference materials is provided.
( ) 3. Supplementary reference materials are provided emphasizing application of mathematics to industrial, business, or home situations.
( ) 4. Teacher-prepared materials, such as study guides, are provided.
( ) 5. Graphic charts and diagrams are available.
( ) 6. Motion pictures, filmstrips, and slides are available.
( ) 7. Files of materials illustrating applications of mathematics are provided.
( ) 8. Other.
( ) 9. Other.

EVALUATIONS

( ) *a. How adequate is the* variety *of instructional materials?*
( ) *b. How adequate is the* content *of instructional materials?*
( ) *c. How effectively are pupils guided in the use of instructional materials?*

D. Methods of Evaluation

CHECK LIST

( ) 1. Evaluation is an integral part of the teaching-learning activities.
( ) 2. The types of evaluation activities selected are determined by the objectives to be measured.
( ) 3. Evaluation activities include interpretation of mathematical concepts as well as operational skills.
( ) 4. Tests are used to determine the mathematical aptitudes of pupils.
( ) 5. Tests are used to determine the mathematical achievement levels of all pupils.
( ) 6. Evaluation activities identify pupils needing remedial activities.
( ) 7. Evaluation activities measure the use of mathematics in functional situations.
( ) 8. Interpretation of test results is used in conjunction with other information to assist pupils in the election of further mathematics activities.
( ) 9. Teacher-made objective tests are used.
( ) 10. Interpretation of evaluation activities provides a basis for planning further instruction.
( ) 11. Pupils are encouraged to evaluate their own achievement.
( ) 12. Both teachers and pupils recognize that tests should be used to reveal strengths and to point out areas for improvement.
( ) 13. Other.
( ) 14. Other.

EVALUATIONS

( ) *a. How comprehensive are evaluation activities in mathematics?*
( ) *b. To what extent do teachers use evaluation results in analyzing the effectiveness of their teaching?*
( ) *c. To what extent do evaluation procedures help the pupil understand the nature of his growth in mathematics?*
( ) *d. To what extent do evaluation procedures identify pupils of unusual promise in the field of mathematics?*

OUTCOMES

(No check list items are prepared for this division, since they would be largely repetitions of the check list items in preceding divisions.)

EVALUATIONS

( ) a. *How efficient are pupils in the use of number and computational skills?*

( ) b. *To what degree do pupils possess an understanding and appreciation of fundamental geometric and space relationships?*

( ) c. *To what degree do pupils possess ability to construct and apply a variety of geometric forms?*

( ) d. *To what degree do pupils possess ability to present data in graphic form?*

( ) e. *To what degree do pupils possess ability to understand and interpret graphic representations?*

( ) f. *To what degree do pupils possess knowledge and ability in analyzing quantitative problems?*

( ) g. *To what degree can pupils use and interpret symbolic language?*

( ) h. *To what extent are pupils beginning to comprehend the significance of the mathematical concepts and skills they are using?*

( ) i. *How effectively do pupils make application of mathematics skills in practical situations?*

( ) j. *To what extent do pupils understand the use and importance of mathematics in many phases of living?*

III. Teacher-made Tests

Teachers will find it helpful to build up a file of index cards containing items and ideas suitable for testing purposes. Some of these will be suggested by readings; some may come from pupil-made items. It will also help if these items are rated, on a scale of 1 to 5, as to level of difficulty so that tests as well as instruction may be individualized.

One danger to be avoided is that of making the subject matter more important than the pupil. Factual knowledge is important, but teachers must not be content to limit evaluation to that one type of outcome. How nearly ready are the pupils for intelligent behavior? This implies reflective thought and not just mechanical manipulation of techniques or verbalizing in parrot fashion. Tests, therefore, should contain elements of novelty, of variety. For example, it is customary to say, "Solve for x: $7x - 17 = 39$." Give variety by changing to "For what value of a does $7a - 17 = 39$ indicate?" Or "What number, if multiplied by 7 and that result diminished by 17, gives 39?"

Some examples of possible test items or procedures are as follows:

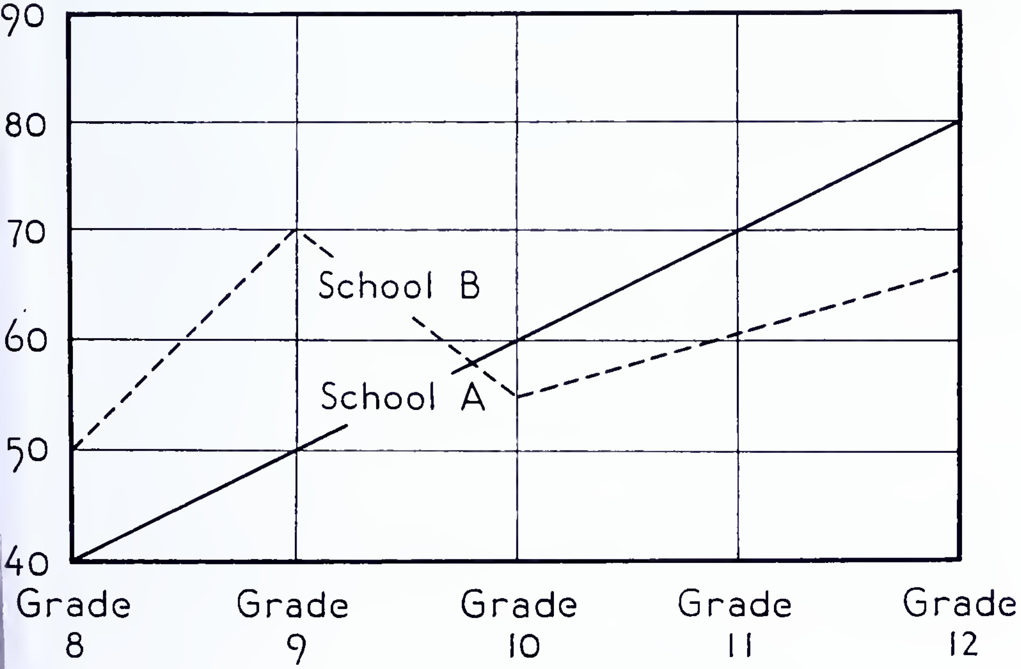
A. *Directions:* The numbers in each of the series below proceed according to some rule. In each series you are to find the next number.

(a)	2	5	8	11	14	17	20	______
(b)	2	4	5	10	11	22	23	______
(c)	20	10	12	6	8	4	6	______

B. The mathematics department of two neighboring towns decided to develop a mathematics test which would be complete enough to measure the mathematical mastery of students from the eighth to the twelfth grades inclusive.

This test was so designed that each student would start at the beginning and work through the test as far as possible. Since the test was created to test the pupil's ability at the various grade levels, it was progressively more and more difficult and contained only subject matter which could be passed after the student had studied the respective courses at the various grade levels.

The chart below shows the percentage of total items which the various grades had correct on this test for the two schools.



From the above paragraph and the chart you can obtain sufficient data to respond to statements 1 through 8. At the left of each statement write in the appropriate letter chosen from the following key:

a. If the statement is completely true.
b. If the statement is probably true.
c. If there is insufficient evidence to determine whether the statement is true or false.
d. If the statement is probably false.
e. If the statement is completely false.

_______ 1. A pupil who attends school A can expect a constantly increasing score on this test as he progresses from the eighth to the twelfth grade.

_______ 2. A pupil who attends school B can expect lower scores on the test in the upper grades than in the lower grades.

_______ 3. A wise parent would send his child to school B for his eighth and ninth grade work but transfer him to school A for his tenth, eleventh, and twelfth grades.

_______ 4. Teachers in the tenth and twelfth grades of school B have less education than the teachers of the same grades in school A.

_______ 5. Pupils attending school A have higher I.Q.'s than those attending school B.

_______ 6. The mathematics teachers of school B are not giving the pupils the fundamentals they need to do the work in the higher grades.

_______ 7. Pupils in school A in the twelfth grade average twice as much as they do in the eighth grade.

_______ 8. This information goes a long way to support the theory that the I.Q. does change.

C. Pass around suitable clippings or pictures and ask pupils to note the mathematical principle involved. A variation of this is to show an example of advertising and have pupils write down the underlying assumption behind the "selling line."

D. To test for recognition of the misuse of mathematical terms and concepts, present a statement in which the pupil is asked to underline the word or words used incorrectly. Example: To find the height of a tower, a surveyor measures the angle of elevation from a selected spot and then moves fifty yards toward the tower. He then measures the angle of elevation from the new spot. The determination of this *smaller* angle will give him sufficient data to determine the height of the tower.

E. Problems in life have more than the necessary data. The ability to sift the significant from the extraneous can be noted by using word problems in which nonessential data are also included.

F. To see whether or not mathematical manipulations are understood, give an example (such as $2x = 14$; $x = 7$.) and ask pupils to state the mathematical principle or generalization which was involved in the process.

IV. Observation

A. In board work or in group work does the pupil who is being evaluated usually discover errors in the work of others?

B. Is he critical of his own accuracy in computation and measuring?

C. Does he habitually check his problems? Can he test the reasonableness of his answers?

D. Does he realize that measurement is used in almost every vocation?

E. Has he improved in his skill to construct geometric figures?

F. Can he readily make generalizations after a few specific examples have been presented?

G. Does he show a growing interest in mathematics? Does he voluntarily bring to class pictures, articles, puzzles, anecdotes of mathematical import?

H. Does he have a clearer picture of his life's goals: (a) vocationally? (b) as a responsible citizen? (c) as an intelligent consumer?

V. Pretests

Pretests should be given to locate the areas of ignorance or misunderstanding before a unit is taught. If certain aspects are already known, time may be more effectively spent on the other matters.

VI. Culminating Activities

(Usually at the end of a unit to tie together the many activities.)

A. A display for other pupils and parents to see. Posters, pamphlets, playlets, debates, forums might well form the nucleus for a display on "Getting the most for your tax dollar."

B. Understandings as revealed by comments in debates or in forums on problems such as "Shall I rent or buy a home?"

C. Individual booklets (see sample items in geometry outline about geometry in advertising.) Look for originality, neatness, accuracy, correct perception of mathematical applications.

D. Individual reports on themes such as "The Mathematical Training Necessary for the Vocation of My Choice."

VII. Work in the Mathematics Laboratory

A. Has he made a workable transit?

B. Does he develop alternate proofs in geometry?

C. Does he do creative work in designs?

D. Does he keep a list of principles studied and make an original note of a practical application of each principle?

VIII. Pupil's Self-Check List

With each unit a set of self-appraisal exercises may be developed to motivate self-analysis and also serve as an overview of the unit. Items for this list may be made by teachers and pupils working together, thus further reviewing the significant aspects.

PUPIL SELF-APPRAISAL CHECK LIST

		CHECK	
	Yes	*No*	*D.*[1] (or Sometimes)
A. Sample Questions			
1. Can I interpret graphs which appear in our daily newspaper?			
2. Do I look for assumptions behind controversial statements which my classmates make?			
3. Can I analyze verbal statements and translate them into the shorthand of a mathematical equation?			
4. Can I locate the irrelevant material in a problem or in an argument?			
5. Can I represent three-dimensional figures of a plane surface?			
6. Do I know how to use			
(a) a metric rule?			
(b) a vernier caliper?			
(c) a micrometer?			
(d) a slide rule?			

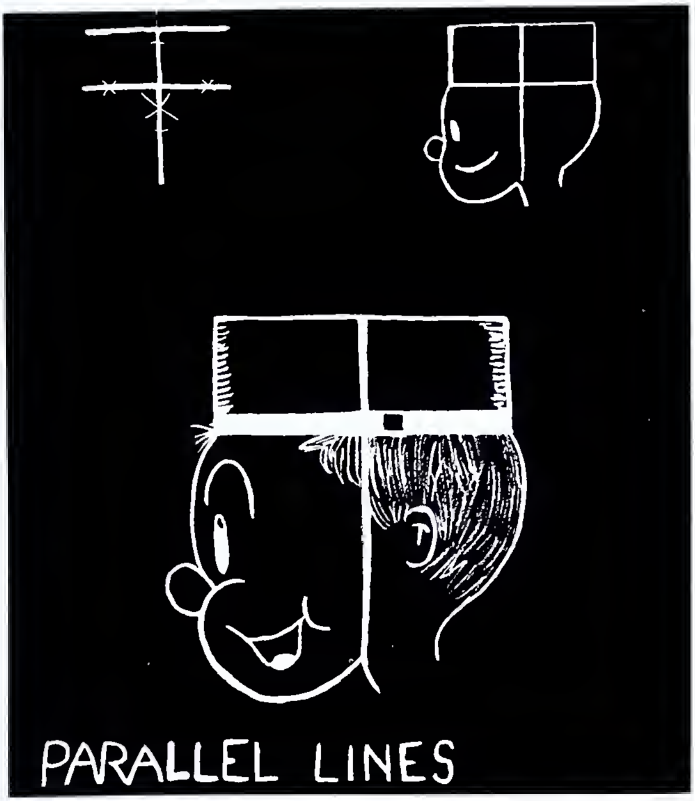
B. Interest inventories

C. Progress charts on assignments and/or tests

D. Sociograms (Although these are made by individual pupils, they need to be studied by trained teachers.)

---

[1] Doubtful.



GEOMETRICAL CONSTRUCTION AIDS IN MAKING CARTOONS

IX. Follow-up Studies

A. How well are they (those who have been in your classes and have graduated one, five, ten years ago) making out in the community?

B. Do managers of business and industry say that these pupils are mathematically competent?

C. How well have they succeeded in college?

X. Evaluating Student Achievement

1. *Pupil Self-appraisal*

The following self-appraisal student rating chart, which is based on the Ten Imperative Needs of Youth, may be used in a class or homeroom: (1) to stimulate needed self-analysis, (2) to call attention of students to areas of personal growth, (3) to recognize appropriate areas and (4) to evaluate progress. A later checking by the pupil may show areas of growth. The items may be changed to suit local conditions when the chart is mimeographed.

WHAT KIND OF PERSON ARE YOU BECOMING?

(Form to be mimeographed)

What kind of person a youth becomes is as important as what he knows. The school is concerned with each student's growth as an individual. This chart is intended to help to show you where you are and how you can improve. You should carefully rate yourself, then the teacher will add his rating. A conference may follow.

________________	________________	________________
Name	*Homeroom*	*Section*

Mark an X covering each equare which best describes how you think, feel, or act with respect to the need which is in the first column.

	1	2	3	4	5
NEEDS OF YOUTH	DOUBTFUL	PASSIVE	PRODUCTIVE	CONSTRUCTIVE	CREATIVE
WORK	Gives up Careless Shuns work	Dependent Submissive Follows others	Interested Loyal Leads sometimes	Definite Strong Often leads	Diligent Confident Makes things go
HEALTH	Slovenly Depressed Stolid	Neat Willing Promising	Healthy Alert Active	Brisk Cheerful Vigorous	Exuberant Enthusiastic Vitalizing
(School) CITIZENSHIP	Unsound Annoying Critical	Indifferent Unconcerned Conforms	Interested Loyal Careful	Devoted Eager Dependable	Stimulating Inspiring Influential
HOME (Room)	Petty Anxious Impatient	Rough and ready Thoughtless Easy-going	Cheerful Deliberate Cooperative	Polite Tactful Warm-hearted	Successful Poised Gracious
THRIFT	Frivolous Buys on whim Wastes time	Muddled Gullible Impulsive	Serious Knows values Purposeful	Sound Keen bargainer Determined	Reliable Brilliant trader Independent
SCIENCE	Opinionated Irrational Prejudiced	Credulous Confused Snap judgment	Open-minded Seeks proof Sticks to facts	Curious Systematic Weighs evidence	Inductive Rational Keen
APPRECIATION	Profane Common Critical of best	Superficial Erroneous Coarse	Knows form Has good taste Recognizes merit	Enjoys form Refreshing Refined	Artistic Creative Elegant
LEISURE	Cheap taste Childish fun Rough	Underbred Follows others Negligent	Restrained Normal pursuits Respects the best	Good hobbies Has some sport Many-sided	Ingenious Mature tastes Enjoys art, music
SOCIABILITY	Self-centered Timid Rude	Indifferent Passive Proper	Accepted Cheerful Sympathetic	Sought Active Helpful	Esteemed Magnetic Influential
LANGUAGE	Careless Peculiar Dormant	Unreliable Deficient Developing	Accurate Practical Passable	Skillful Forceful Desirable	Colorful Persuasive Delightful

Credit yourself 1, 2, 3, 4, 5,

2. *A Pupil's Points of View Concerning the Learning Situation*

The learner's reaction determines his participation. An evaluation of pupil reaction can (1) provide direction for curriculum improvement, (2) reveal causes of pupil frustration or aggression, and (3) create objective points of view on the part of pupils which foster greater personal efforts.

POINTS OF VIEW CONCERNING YOUR COURSE

(Form to be mimeographed)

Here is a list of statements about your course. What do you think is good and what not so good You are asked to check the statements—under *Yes, Doubtful,* or *No*—so that there may be a good report on what you think. You need not sign your name.

Check each statement truthfully and thoughtfully so that the best kind of course can be planned for you and for those who follow you.

		CHECK	
Examples:	*Yes*	*D*[1].	*No*
The topics I study in this subject are very valuable to me.	✓		
My fellow students are friendly.	✓		
My teacher is hard to get to know.		✓	

	Yes	*D.*	*No*
1. Most of my work in this class is done only to get a good grade.			
2. I would be learning more if I were working somewhere.			
3. This subject is "over my head."			
4. The teacher in this class does most of the talking.			
5. Most of my work is assigned by the teacher.			
6. I am afraid when I am called on to recite or report.			
7. There are many disciplinary cases in this class.			
8. The teacher seems to like his job.			
9. The teacher has a good sense of humor.			
10. The teacher praises pupils more often than he blames them.			
11. The teacher takes an interest in me.			

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[1] Doubtful.

		CHECK	
	Yes	*D*[1].	*No.*
12. The teacher mixes freely with the pupils.			
13. The teacher seems "up to date" in what he knows and does.			
14. I feel free to talk over my personal problems with this teacher.			
15. I need help in studying this subject.			
16. This course will help me to become the kind of person I want to be.			
17. The pupils help to plan the work in this class.			
18. This course is about problems and ideas that interest me.			
19. I am getting experiences in this class that will be valuable all my life.			
20. My work is teaching me to think—to discover facts and make decisions.			
21. I am learning how to work with other people in groups and on committees.			
22. I get a chance to act as a leader in this class.			
23. I am learning how to get ideas from my community and from the library.			
24. I would like to have time to do still more work in this course.			
25. I am enthusiastic about my work in this class.			

If you were planning for this class, what changes would you make?

List any changes. Use the other side of this paper.

3. *Reporting Pupil Progress*

Tradition and facility in pupil accounting favor the retention of subject marks and Carnegie Units in secondary school record keeping. Yet parents, when questioned widely, have given maximum importance to the development of the pupil's personality.[1] What is evaluated determines what is taught. Logically, then, both subject marks and items appraising growth in desirable types of behavior should have place and status on report cards.

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[1] Doubtful.

Many school faculties, through cooperation with parents, have developed comprehensive means of reporting pupil progress. The report cards used indicate subject marks on one side and ratings of growth in desirable behaviors on the other. The exactness of the behavior ratings is secondary to the status which is given to them. The following is an example and a suggestion:

Form for back of report cards

RECORD OF PERSONAL DEVELOPMENT

The pupil should rate himself; then the parent will add a rating; and then the teacher. Key: 1 ___ a little; 3 ___ below average; 5 ___ average; 7 ___ above average; 10 ___ excellent.

NEEDS OF YOUTH	Description of Desirable Behaviors	Achievement Ratings by		
		Pupil	Parent	Teacher
1. WORK	Works hard; work done on time Working toward a life career			
2. HEALTH	Has good health habits Keeps self clean and neat			
3. CITIZENSHIP	Uses freedom wisely Helps to make things better			
4. HOME	Helps in work at home Strives to keep home happy			
5. THRIFT	Earns and saves money Spends money wisely			
6. SCIENCE	Uses good judgment Can explain phenomena			
7. APPRECIATION	Reads good literature Enjoys art and music			
8. LEISURE	Enjoys school activities Has a sport or hobby			
9. SOCIABILITY	Has many friends Is courteous to all			
10. LANGUAGE	Thinks logically Writes and speaks well			
	Total			

Notes:
1. Similar rating sections may be added for each marking period.
2. School organization should provide at least one teacher who knows the pupil well.
3. Experience has shown a high degree of parent approval and cooperation.

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[1] "What U. S. Thinks About Its Schools," *Life*, Volume 29, Number 16, October 16, 1950.

XI. Study of Dropouts

Whether or not it is a school practice to investigate dropouts, each mathematics teacher should join in making such a study. He should ask himself such questions as: Why did they leave school? Did my teaching contribute to their inability to discern the relationship between mathematics and future occupations? Were they failing because I failed to teach them how to study? Were they dissatisfied because I failed to individualize my instruction? Did I seek to find their real needs and attempt to develop a suitable program in mathematics?

What Any Supervisor May Ask About Learning Activities in Any Class

Planning

Do the learning activities give evidence of careful and intelligent planning on the part of the teacher? Is any opportunity afforded the learners to participate in the planning?

Objectives

Are the learning activities directed to the accomplishment of objectives (understandings, skills, or attitudes) which are definite, clear-cut,— no doubt as to what they are, specific (capable of achievement within the class period), and worthy (related directly to the actual or probable needs of youth, community, society)?

Learning Activities

1. Is there maximal pupil participation in the learning activities? (Law of self-activity)
2. Are the students actively interested in the learning activities? (Law of felt need—motivation)
3. Do the learning activities take into consideration the wide range of pupil differences in ability, interests, experiences, purposes, etc.? (Provision for individual differences)
4. Do the learning activities make use of appropriate audio-visual aids? (Combination of stimuli—use of blackboard, posters, demonstrations, excursions, movies, records, models, etc.)
5. Are there sufficient variety and appropriate timing of the learning activities?
6. Are the learning activities sprinkled with illustrations within the experiences of the learners? (Law of association)
7. Are good pupil responses praised? (Law of effect)
8. Are the learning activities lifelike? (Law of transfer)
9. Are the learners succeeding with the learning activities? Can they see their own progress? (Evaluation)

Classroom Conditions

1. Is the classroom attractive?
2. Is adequate attention paid to lighting, seating, and ventilation?
3. Does the classroom atmosphere show any relationship to the learning activities being carried on in it?

Classroom Relationships

1. Are the pupil-teacher, teacher-pupil, and pupil-pupil relationships satisfactory?
2. Are the classroom relationships characterized by a mental hygiene point of view and by a democratic spirit?
3. Do the learners conduct themselves well?
4. Is discipline indirect and voluntary or direct and forced?

Assignment

1. Are the learning activities to be undertaken defined clearly and motivated fully? Do pupils help to define the learning activities?
2. Are the assignments which are to be done outside class appropriate in nature and extent?

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A Prayer for Teachers

O LORD of Learnings and of Learners, we are at best but blunderers in this godlike business of teaching. We have been content to be merchants of dead yesterdays, when we should have been guides into unborn tomorrows. We have put conformity to old customs above curiosity about new ideas.

We have been peddlers of petty accuracies, when we should have been priests and prophets of abundant living. We have thought more about our subject than our object. We have schooled our students to be clever competitors in the world as it is, when we should have been helping them to become creative cooperators in the making of the world as it is to be.

We have counted knowledge more precious than wisdom. We have tried to teach our students what to think instead of how to think. We have thought it our business to furnish the minds of our students, when we should have been laboring to free their minds.

It has been easier to tell our students about the motionless past that we can learn once for all, than to join with them in trying to understand the living present that must be studied afresh each morning. From these sins of sloth may we be freed. May we realize that it is important to know the past only that we may live wisely in the present. Help us to be more interested in stimulating the builders of modern cathedrals than retailing to students the glories of ancient temples.

Give us to see that a student's memory should be a tool as well as a treasure chest. Help us to realize in the deepest sense, we cannot teach anybody anything; that the best we can do is to help them learn for themselves.

Save us from the blight of specialism; give us reverence for our materials, that we may master the facts of our particular fields, but help us to see that all facts are dead until they are related to the rest of the knowledge and to the rest of life. May we know how to relate the "coal scuttle to the universe."

Help us to see that education is, after all, but the adventure of trying to make ourselves at home in the modern world. May we be shepherds of the spirit as well as masters of the mind.

Give us, O Lord of Learners, a sense of the divinity of our undertaking. GLENN FRANK.

CHAPTER V

SUPPLEMENTARY MATERIALS

FREE AND INEXPENSIVE MATHEMATICS TEACHING AIDS

Prepared by the Special Projects Committee for the National Council of Teachers of Mathematics Meeting at Chicago April, 1950

1. American Bankers Assn., 12 E. 36th St., New York 16, N. Y.
   Personal Money Management, H. M. Hart. 31-page booklet, price 25 cents. Supplementary material in budgeting, written in style suitable for high school students
2. American Can Co., Sales Promotion Division, 230 Park Ave., New York 17, N. Y.
   A Word About Tin Cans; High School Manual on Commercially Canned Foods. Free pamphlets
3. American Optical Co., Scientific Instrument Division, Buffalo 15, New York
   "Optical Performance Predicted on Paper." A poster for bulletin board use. Free
4. Baltimore & Ohio Railroad, Baltimore 1, Maryland
   Railroad 'Rithmetic, Book I; Railroad 'Rithmetic, Book II. Book I contains problems suitable for elementary school level, Book II for high school level. Free
5. Bausch & Lamb Optical Co., Rochester, New York
   Milestones in Optical History. Contains 12 very attractive historical pictures in color each with a description of the subject. Suitable for bulletin board use or for framing. Free
   Educational Materials. Mimeographed list of materials supplied by the company, some of which are free
6. Bell Telephone Laboratories, 463 West St., New York 14, N. Y.
   "It Tells How Often You Will Telephone." A poster for bulletin board use. Free
7. Better Vision Institute, Suite 3157, 630 Fifth Avenue, New York 20, N. Y.
   Galileo; Leeuwenhoek, Miscroscope Maker; Talbot and Daguerre, Inventors of Photography. Free. Very small leaflets. The above are 3 of 30 in the Men of Vision Science Classic Series
8. Boston Better Business Bureau Educational Division, Boston, Mass.
   Facts You Should Know About—Borrowing, Budgeting, Saving, Legal Problems, Commercial Banks and Trust Companies, Investment Companies, and Life Insurance. Seven booklets, price 5 cents each. Excellent background material for the units named above
9. Bruning Co., 4700 Montrose Avenue, Chicago 41, Illinois
   "Decimal Equivalent Chart." Free
10. Building America, 2 W. 45th St., New York 19, N. Y.
    Taxes. 31-page booklet, price 30 cents. Published for the NEA Department of Supervision and Curriculum Development. Well illustrated and excellent for classroom reading table
11. Bureau of Pubications, Teachers College, Columbia University, New York, N. Y.
    Numbers and Numerals. D. E. Smith and J. Ginsburg. Book, price 25 cents. Supplementary reading
12. Burroughs Adding Machine Co., 6071 Second Ave., Detroit 32, Michigan
    Fascinating Figure Puzzles. Small booklet compiled by F. W. Davis, good for all classes. Free

The Story of Figures. Free booklet. Good for reading table or clubs. Junior and senior high school classes

13. Canadian Mathematical Congress, Engineering Bldg., McGill University, Montreal, Canada
    Why Study Mathematics. 33-page booklet prepared for use of teachers and students. Price 50 cents
14. J. I. Case Co., Racine, Wisconsin
    Level Farming on Sloping Fields. A booklet, free
15. Cooperative Recreation Service, Delaware, Ohio
    Puzzle Craft, Kit U. Price 25 cents. Plan for making and solving 40 puzzles in wire, wood, and string. Good for clubs
    Brain Resters and Testers, Kit M. 25 cents. A booklet of puzzle problems
16. Division of Surveys and Field Services, George Peabody College for Teachers, Nashville, Tenn.
    Free and Inexpensive Learning Materials. A catalogue, *not* limited to mathematics aids. Price 25 cents
17. Dunn and Bradstreet, Inc., Education Division, New York City
    "Recent Trends in Production, etc."
    "Quarterly Retail Sales and Inventories"
    "Wartime and Postwar Shifts in U. S. Populations"
    "Commercial Failures in U. S., 1857 to Date." Good bulletin board material in teaching graphs in junior and senior high school. Other charts also available, free
18. Education Section, U. S. Savings Bonds Division, Treasury Dept., Washington, D. C.
    Lessons in Arithmetic Through School Savings
    Teaching Mathematics Through School Savings
    Budgeting for Security. Available free in any quantity desired. Useful for arithmetic and general mathematics
19. Federal Reserve Bank of Minneapolis, Minneapolis, Minn.
    Your Money and the Federal Reserve System. 30-page illustrated book for general mathematics. Free
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21. Fogelson, Ida D., 5520 South Shore Drive, Chicago 37, Ill.
    "Basic Laws of Mathematics." Addition-Subtraction-Multiplication-Division. Four pictures in color, ready for framing. Price of four—50 cents, postpaid.
22. Ford Motor Co., Education Dept., Dearborn, Mich.
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23. Ford, Lester R., Illinois Institute of Technology, S. Federal St., Chicago, Ill.
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29. Henry Holt Co., 600 W. Van Buren St., Chicago 7, Ill.
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32. Johnson, Donovan (University of Minnesota) and Syer, H. W. (Boston University)
    A Bibliography of Mathematics Films and Filmstrips. Reprint from *School Science and Mathematics,* Nov., 1949

33. Jones, Phillip S., University of Michigan, Department of Mathematics, Ann Arbor, Michigan
    "How High School Mathematics Can Contribute to Your Career." Poster, price 10¢

34. Joseph, Margaret, 1504 N. Prospect Ave., Milwaukee 2, Wis.
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35. Los Angeles City School District, Curriculum Division, Office of Mr. Dale Carpenter, 1205 W. Pico Blvd., Los Angeles 15, California
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51. Syer, Henry, Boston University School of Education, 84 Exeter St., Boston, Mass.
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52. Teaching Aids Service, New Jersey State Teachers College, Upper Montclair, New Jersey
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53. The Travelers' Insurance Co., 700 Main St., Hartford 15, Conn.
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54. United Air Lines, Palmer House, Chicago, Ill.
    Air Transport Facts and Figures. Free booklet, useful in teaching unit in graphs

55. U. S. Department of Commerce, Civil Aeronautics Administration, Office of Aviation Training, Aviation Education Division, Washington 25, D. C.
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56. U. S. Department of Commerce, Coast and Geodetic Survey, U. S. Government Printing Office, Washington, D. C.:
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    D-405—"Melancholy." Dürer
    Prints, size 5½″ by 8″, price 2 cents each

58. Washington Service Bureau, 1217 Thirteenth St., N. W., Washington, D .C.
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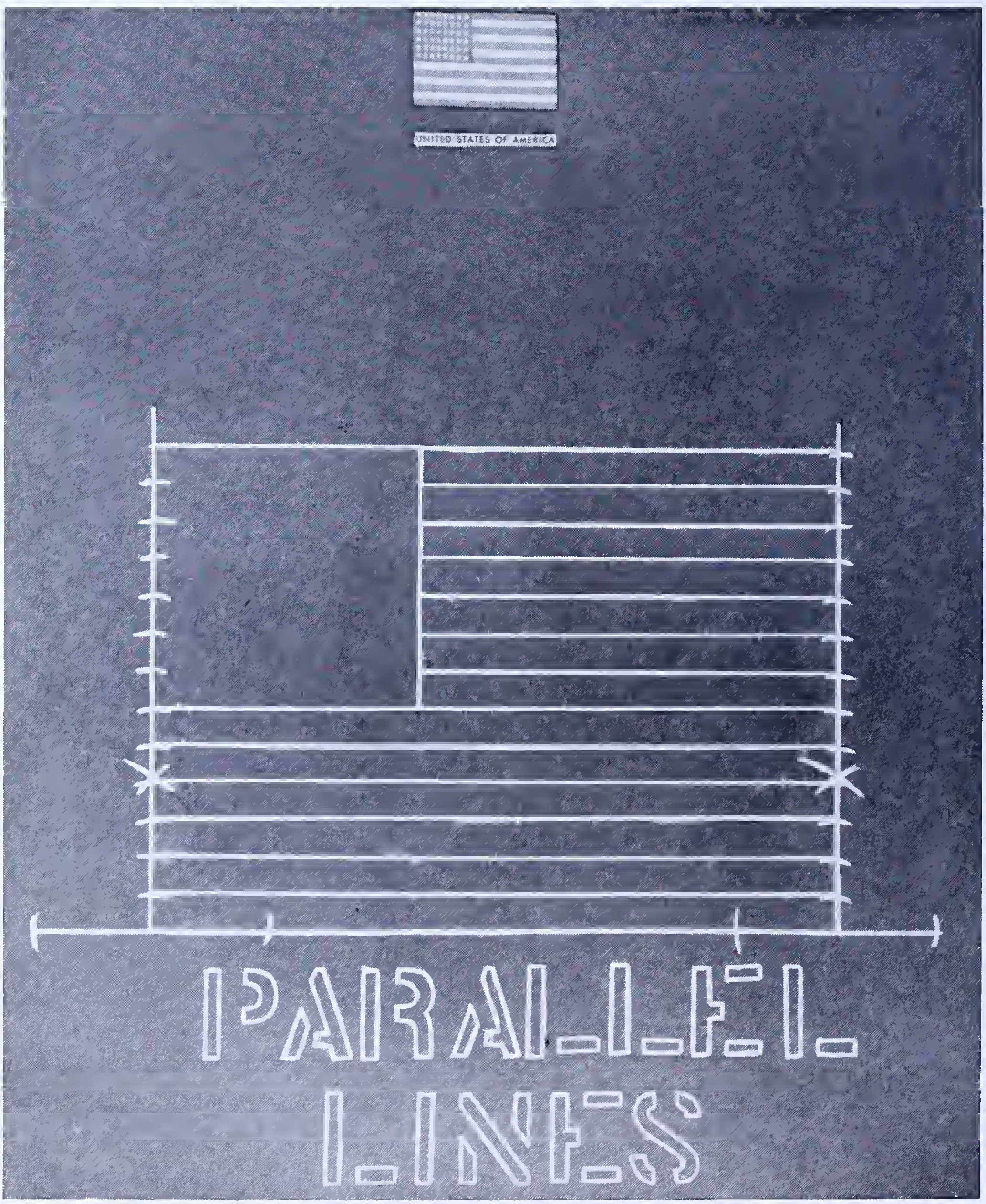
APPENDIX

1. Results of an Inquiry on the Needs of Youth

2. Partial Listing of Some Vocations Which Require Mathematics

3. Acknowledgments


UNITED STATES OF AMERICA
PARALLEL
LINES

APPENDIX

1.

RESULTS OF AN INQUIRY ON THE NEEDS OF YOUTH

In order that mathematics teachers may have data readily available, the following survey results are reproduced from Bulletin 243, *Curriculum Impovement by a Secondary School Faculty*. The Ten Imperative Needs of Youth are similar to those included in Section C of the 1950 *Evaluative Criteria* of the Cooperative Study of Secondary School Standards. See Chapter IV, "Evaluation." The Study proposes that:

> "It is generally accepted that the main purpose of secondary education in American democracy is to meet the educational needs of all youth of secondary school age. . . . Youth have both common and individual needs. . . . Statements (of needs) are, in reality, objectives for secondary schools. . . ."

Summaries from nearly 5,000 Pennsylvania youth, grades seven through twelve, are shown in percentages. Areas are shown in which a more functional type of mathematics education has a great opportunity.

A test-retest determination of the group reliability of the questionnaire showed a correlation coefficient of .968. After an interval of ten days, group responses were highly consistent with those originally made. A correlation of .932 between junior high school and senior high school responses indicates that stress upon verbal ideas alone does not meet the basic needs of youth. Meeting pupil problems and providing basic knowledge for living require careful planning and functional teaching.

I. WORK

All youth need to develop skill as workers and those understandings and attitudes that fit the worker to earn money.

	PERCENTAGES		
	Yes	*No*	*D.*[1]
A. Do you feel the need for more information about jobs or professions?	62	32	6
B. Do you have success more often than failure in your school work?	73	19	8
C. Do you know for what kind of jobs each of your school subjects can help to prepare you?	53	37	10
D. Do you need, for your future life work, some course that your school does not give?	32	52	16
E. Do you know about local job opportunities, requirements and pay?	59	33	8

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[1] Doubtful.

II. HEALTH

All youth need to develop and maintain good health and physical fitness.

		PERCENTAGES		
		Yes	*No*	*D.*
A.	Can you apply first aid and artificial respiration?	48	38	14
B.	Are you as happy as other people seem to be?	69	22	9
C.	Are you worried about your own growth or health?	39	56	5
D.	Are you developing skill in any sport which you can follow after you leave school?	42	50	8
E.	Do you know how to select a well-balanced diet of carbohydrates, proteins, minerals, vitamins, etc.?	50	39	11

III. CITIZENSHIP

All youth need to understand the right of the citizen of a democratic society and their duties as members of the community and citizens of the State and nation.

		PERCENTAGES		
		Yes	*No*	*D.*
A.	Do you feel that other students do not like you as well as you wish they did?	68	19	13
B.	Do you feel at ease in talking or working with people of another color, race, or creed?	82	19	1
C.	Would you like to share with your classroom teachers in planning your school work?	72	19	9
D.	Can you compare the freedoms of the American way of life with those of Communism and Socialism?	46	33	21
E.	Have you seen a court or other public officials when they were at work?	28	70	2

IV. HOME

All youth need to understand the place of the family for the individual and society and what helps in successful family life.

		PERCENTAGES		
		Yes	*No*	*D.*
A.	Can you describe the purposes of the family in American life?	33	62	5
B.	Do you feel that there is lack of understanding between you and your parents or between you and your brothers and sisters (if any)?	29	65	6
C.	Can you read the meters—gas, water, or electric—in your home?	36	44	10
D.	Do you need to learn about making dates or choosing a mate or making a home?	45	42	13
E.	Do you understand your own sexual growth and its effect on what you feel and do?	69	18	13

V. THRIFT

All youth need to know how to buy and use goods and services intelligently and to understand both the values received and the consequences of their acts.

	PERCENTAGES		
	Yes	*No*	*D.*
A. Do you know how to pick the best quality in the food and clothes you buy?	62	31	7
B. Do you know where to find information about how and what to buy?	63	29	8
C. Can you tell if what is said in advertising or politics is true?	48	38	14
D. Are you, yourself, ever asked to judge the value and success of your school work or personal development?	52	36	12
E. Do you know how to invest money—buy stocks and bonds, etc.?	36	59	5

VI. SCIENCE

All youth need to understand the scientific method, the influence of science on human life, and the main scientific facts concerning the nature of the world and of men.

	PERCENTAGES		
	Yes	*No*	*D.*
A. Do you need to know how to work out practical problems of your own, gathering facts and information, judging the facts, knitting them together, and coming to your own conclusions?	65	21	14
B. Do you see how your work in science is connected with your own problems?	77	16	7
C. Do you understand people—what makes most people think, feel and act as they do?	53	36	11
D. Do you understand how motors, toasters, thermometer, etc., actually work?	47	44	9
E. Can you describe some problems which science has brought along with its benefits?	54	32	14

VII. APPRECIATION

All youth need to learn how to appreciate beauty in literature, art, music, and nature.

	PERCENTAGES		
	Yes	*No*	*D.*
A. Is your school a beautiful place in which to live and learn?	71	20	9
B. Do you know the names of adult groups in your community which promote the study of literature, arts, music, or nature?	34	57	9
C. Do you believe that you are learning to enjoy better art, better music, and better literature as you go on with your education?	73	21	6
D. Have you been on any trips to art museums or concerts or for nature study?	44	33	23
E. Do you read the excellent fiction and other literature which is related to your school subjects?	40	49	11

VIII. LEISURE

All youth need the ability to use their leisure time well and to divide it wisely, balancing activities that yield satisfaction to the individual with those that are socially useful.

	PERCENTAGES		
	Yes	*No*	*D.*
A. Does your community provide enough places for wholesome recreation?	35	57	8
B. Do you feel that you are "left out of things" in school?	20	71	9
C. Would you like to work on some community welfare activities such as the Red Cross drive, in your leisure time?	54	33	13
D. Are you doing any work on what may be your life-time hobbies?	47	47	6
E. Is your leisure time being spent as you hope your own children will spend their leisure time?	47	36	17

IX. OTHER PEOPLE

All youth need to develop respect for other persons, to grow in their insight into values and principles, and to be able to live and work with others.

	PERCENTAGES		
	Yes	*No*	*D.*
A. Do you know how to be a good committee chairman?	27	56	17
B. Do you feel the need to know more about introducing people, about courtesy, etc.?	55	37	8
C. Do you wish that your teachers would like you better?	51	31	18
D. Do you prefer to be by yourself rather than with other students?	8	82	10
E. Do you feel the need to learn more about getting along with people?	46	42	12

X. LANGUAGE

All youth need to grow in their ability to express their thoughts clearly and to read and listen with understanding.

	PERCENTAGES		
	Yes	*No*	*D.*
A. Do you use good written and spoken English in all of your classes?	48	39	13
B. Do you wish that you knew how to study better?	81	14	5
C. Are you developing the habit of daily newspaper reading?	74	23	3
D. Do you know how to find reference material by using the card catalogue, the Reader's Guide, etc., in the school library?	72	23	5
E. Can you write a good letter or composition?	42	38	20

CODE

M—Much
S—Some
L—Little

PARTIAL LISTING OF SOME VOCATIONS WHICH REQUIRE MATHEMATICS

NAME OF VOCATION	TRAINING REQUIRED					D. O. T. No.[1]	TYPE OF SKILL REQUIRED			MAJOR EMPHASIS			BRIEF DEFINITION
	Graduate	College	Tech. School	High School	Special (on job)		Manual	Academic	Technical	Biology	Chemistry	Physics	
A. Agriculture													
1. Abattoir Work				X	X		M	S	S	S	L	L	Slaughter and Meat Packing Workers
2. Agronomist	X	X		X		0-35.01	S	M	M	M	M	S	Soil and Crop Expert
3. Apiarist		X		X	X	3-07.70	M	S	M	M	S	S	Bee Keeper
4. Arboriculturist		X		X		0-35.05	M	M	M	M	M	S	Tree Expert
5. Author (Scientific)	X	X		X	X	0-06.85	S	M	M	M	M	M	Writes Stories and Articles about Science
6. Dairy Farmer		X		X	X	3-04.10	M	S	S	M	M	S	Supervises Dairy Farm
7. Fisherman				X	X	3-87	M	S	S	S	S	L	Gathers Salable Products in Fresh or Salt Water
8. Forester	X	X		X		0-35.07	M	M	M	M	M	M	Cares for and Manages Forests
9. Fruit Worker				X	X	3-15.22	M	S	L	S	L	L	Picks Fruit. Operates Grader and Packers
10. Fur Trapper				X	X	3-97.30	M	S	M	M	L	L	One Who Traps Fur Bearers for a Living
11. Game Protector		X		X	X	0-35.10	S	M	S	M	S	L	Protects Wildlife
12. Gardener					X	3-40.01	M	L	L	S	L	L	Cares for Flowers and Shrubs on Estates
13. Nurseryman		X		X	X	3-38	M	L	S	M	M	S	Raises Trees and Shrubs Commercially
14. Livestock Farmer		X		X	X	3-07.10	M	L	S	M	M	L	Raises Animals for Food and Food Products
15. Pomologist		X		X		0-35.05	M	M	M	M	M	S	Fruit Specialist
16. Poultryman		X		X	X	3-08.10	S	S	M	M	M	S	Manages Hatching and Rearing Poultry
17. Truck Farmer		X		X	X	3-09.10	S	S	M	M	M	L	Vegetable Farmer
18. Veterinarian	X	X		X		0-34.10	M	M	M	M	M	M	Animal Doctor
B. Biologist and Kindred Occupations													
1. Anthropologist	X	X		X	X	0-36.01	S	M	M	M	S	S	Specialist in History of Man's Biol. Devel.
2. Bacteriologist	X	X		X		0-35.33	S	M	M	M	M	S	Studies Kinds and Effects of Bacteria
3. Botanist	X	X		X		0-35.23	S	M	S	M	S	L	Plant Specialist
4. Cytologist	X	X		X		0-35.22	M	M	M	M	M	M	Studies Structure and Function of Living Cells
5. Ecologist	X	X		X		0-35.23	S	M	M	M	S	L	Studies Life and Its Relation to Its Environment

[1] Refers to
Dictionary of Occupations
Part I—Definitions

CODE
M—Much
S—Some
L—Little

PARTIAL LISTING OF SOME VOCATIONS WHICH REQUIRE MATHEMATICS
(Continued)

	NAME OF VOCATION	TRAINING REQUIRED						TYPE OF SKILL REQUIRED			MAJOR EMPHASIS			
		Graduate	College	Tech. School	High School	Special (on job)	D. O. T. No.[1]	Manual	Academic	Technical	Biology	Chemistry	Physics	BRIEF DEFINITION
6.	Embryologist	X	X		X		0-35.2	S	M	S	M	S	S	Specialist in Life Before Birth
7.	Entomologist	X	X		X	X	0-35.30	M	M	M	M	S	S	Insect Specialist
8.	Herpetologist	X	X		X	X	0-35.28	M	M	M	M	S	S	Reptile Specialist
9.	Histologist	X	X		X	X	0-35.41	M	M	M	M	M	S	Specialist in Living Tissues
10.	Ichthyologist	X	X		X	X	0-35.28	M	M	M	M	S	S	Fish Specialist
11.	Mycologist	X	X		X	X	0-35.25	M	M	M	M	M	S	Fungi Specialist—Mushroom Spawn Maker
12.	Oceanographer	X	X		X		0-35.65	S	M	M	M	S	S	Specialist in Geography of Oceans
13.	Ornithologist	X	X		X	X	0-35.28	S	M	M	M	S	S	Bird Specialist
14.	Paleontologist	X	X		X	X	0-36.02	M	M	M	M	M	S	Specialist in Prehistoric Life (Fossils)
15.	Parasitologist (Plant)	X	X		X	X	0-35.31	M	M	M	M	S	S	Parasite Specialist
16.	Pathologist (Plant)	X	X		X	X	0-35.26	M	M	M	M	M	M	Disease Specialist
17.	Taxidermist			X	X	X	0-66.95	M	S	S	M	M	S	Preserves and Stuffs Animals
18.	Physiologist (Plant)	X	X		X		0-39.37	S	M	S	M	S	S	Specialist in Functions of Living Organs
19.	Taxonomist	X	X		X		0-35.23	S	M	S	M	S	S	Specialist in Classification of Living Things
20.	Zoologist	X	X		X		0-35.28	S	M	M	M	S	S	Animal Specialist
21.	Psychologist	X	X		X	X	0-36.17	S	M	M	M	S	S	Mind Specialist
22.	Psychiatrist	X	X		X	X	0-26.19	S	M	M	M	M	S	Specialist in Treatment of Mental Diseases
23.	Biochemist	X	X		X		0-07.02	S	M	M	M	M	S	Specialist in Chemistry of Living Things
24.	Science Equip. Mfg.	X	X	X	X		5, 7, 9-09	M	S	M	S	S	M	Makes and Sells Scientific Apparatus
25.	Collector (Plant Specimens)		X		X	X	3-49	M	S	S	M	M	S	Collects and Sells Nature Specimens
26.	Sanitary Engineer		X		X		0-16	M	S	M	M	M	M	Devises Best Means of Waste Disposal
27.	Irrigation Engineer		X		X		0-16.0	M	S	M	S	S	M	Devises Best Means of Crop Irrigation
28.	Landscape Architect		X	X	X	X	0-03.20	M	M	M	M	M	M	Engineers Contouring and Planting Landscapes
29.	Laboratory Technician			X	X	X	0-50.25	M	S	M	S	M	S	Aids Research and Tests in Science Laboratory
30.	Medical Specialists													
	a. Physician	X	X		X	X	0-26.10	M	M	M	M	M	S	Doctor Who Administers Medicines
	b. Surgeon	X	X		X	X	0-26.18	M	M	M	M	M	M	Doctor Who Specializes in Operating

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[1] Refers to
Dictionary of Occupations

S —Some
L —Little

(Continued)

NAME OF VOCATION	TRAINING REQUIRED					D. O. T. No.[1]	TYPE OF SKILL REQUIRED			MAJOR EMPHASIS			BRIEF DEFINITION
	Graduate	College	Tech. School	High School	Special (on job)		Manual	Academic	Technical	Biology	Chemistry	Physics	
c. Osteopath	X	X	X	X		0-39.96	M	M	S	M	S	M	Heals Illnesses by Manipulation of Body Parts
d. Dermatologist	X	X		X	X	0-26	M	M	M	M	M	S	Skin Disease Specialist
e. Masseur				X	X	0-52.41	M	S	L	S	L	S	Massage Expert
f. Chiropodist		X		X	X	0-52.01	M	S	S	S	L	M	Foot Doctor
g. Oculist	X	X		X	X	0-26	M	M	M	M	S	M	Eye Specialist
h. Orthodontist		X		X	X	0-13.10	M	S	M	M	M	M	Dentist Who Specializes in Corrective Work
i. Neurologist	X	X		X	X	0-26	S	M	M	M	M	M	Nerve Specialist
j. Cardiologist	X	X		X	X	0-26	S	M	M	M	S	S	Heart Specialist
k. Pediatrician	X	X		X	X	0-26	S	M	M	M	M	S	Specialist in Treating Children's Diseases
32. Museum Curator		X		X	X	0-39.03	M	M	M	M	S	M	Manages the Preparation of Museum Dislays
33. Nature Artist			X	X	X	0-04	S	S	M	S	M	M	Paints and Draws Picture of Wildlife
34. Nature Photographer			X	X		0-56	M	S	M	M	S	S	Takes Pictures of Wildlife
35. Pharmacist		X		X	X	0-25	M	S	M	S	M	S	Mixes Chemicals for Medicines
36. Teacher	X	X		X		0-11 and 0-31	S	M	S	M	M	M	Instructs Students in Various Subjects
37. (Nature) Counselor		X		X	X	0-27.15	M	M	S	M	S	L	Conducts Field Trips and Classes in Nature Study in Summer Camp
38. Agricultural Chemist		X	X	X		0-07	M	M	M	M	M	S	One Who Advises on Crop, Fertilizers, Animal Feeds, Tonics, etc.
a. Spray Expert			X	X	X	3-01 and 7-57	M	S	M	S	M	S	Specialist in Insecticides and Fungicides
b. Fertilizer Mixer			X	X	X	4-52.11	M	S	M	L	M	S	Determines Proper Chemicals to be Used in Making Fertilizer
c. Animal Medicine and Tonic Maker		X	X	X	X	4, 6, 8-53	M	S	M	M	M	S	One Who Mixes Chemicals in Animal Medicines and Tonics
d. Hormones Expert		X		X		0-07	S	M	M	M	M	L	One Who Studies Composition and Effect of Hormones on Life
39. Chemical Engineer	X	X		X		0-15.01	M	M	M	S	M	M	Conducts Engineering Projects in Field of Chem.
40. Metallurgist		X		X		0-14	M	M	M	L	M	M	Determines Metallic Content of Ores and Uses of Same
41. Petroleum Chemist		X		X		0-07	S	M	M	L	M	M	Determines Best Methods of Refining and Using Petroleum
42. Research Chemist	X	X		X		0-07	M	M	M	S	M	M	Conducts Research with All Types of Chemicals

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[1] Refers to
Dictionary of Occupations
Part I—Definitions

CODE

M—Much
S—Some
L—Little

PARTIAL LISTING OF SOME VOCATIONS WHICH REQUIRE MATHEMATICS
(Continued)

NAME OF VOCATION	TRAINING REQUIRED					D. O. T. No.[1]	TYPE OF SKILL REQUIRED			MAJOR EMPHASIS			BRIEF DEFINITION
	Graduate	College	Tech. School	High School	Special (on job)		Manual	Academic	Technical	Biology	Chemistry	Physics	
43. Ceramics Expert		X	X	X		0-15.11	M	M	M	L	M	M	Develops Uses of Clay and Clay Products
44. Building Materials Expert			X	X	X	0-99.21	M	S	M	L	M	M	Develops More and Better Uses of Cement, Sand, etc.
45. Glass Maker			X	X	X	4, 6, 8-65	M	L	S	L	S	M	Supervises Making of Glass and Glass Products
46. Auto Mechanic			X	X	X	5-81.010	M	S	M	L	S	M	Repairs Automobile Parts
47. Fuels Expert (Combustion Engineer)		X	X	X	X	0-19.01	S	S	M	S	M	M	Determines Best Use of All Types of Fuels
48. Lubricant Expert		X	X	X	X	0-39	M	S	M	L	M	M	Develops Proper Uses of Oils and Greases
49. Welder			X	X	X	4-85	M	S	M	L	S	M	Joins Metals with Elec. Arc. or Oxy-Acetylene Torch
50. Sanitary Inspector		X		X	X	0-95.16	M	S	L	S	M	L	Inspects Restaurants, Toilets, Pools to Prevent Disease
51. Cabinet Maker			X	X		4-32.100	M	S	M	L	S	M	Builds Articles of Wood
52. Sheet Metal Worker			X	X		4-80	M	S	M	L	S	M	Builds Articles of Metal
53. Mason			X	X		5-24	M	S	M	L	S	M	Uses Plaster, Mortar, Cement, Clay, to Build and Construct
54. Painter			X	X		5-27.01	M	S	M	L	S	M	Paints Wood, Metal, and Masonry Surfaces for Protection and Beauty
55. Explosive Expert			X	X	X	6-52.4	S	S	M	L	M	M	Directs Making and Proper Uses of Explosives
56. Textile Chemist		X	X	X	X	0-07.03	M	S	M	S	M	M	Directs Making, Dyeing and Finishing of Fabrics
57. Dye Expert		X	X	X	X	5-18.01	S	S	M	S	M	S	Develops Making and Proper Uses of Dyes
58. Civil Engineer		X		X		0-16.01	M	M	M	S	M	M	Designs and Directs Construction of Public Facilities
59. Acoustical Engineer		X		X		0-17	M	M	M	S	S	M	Sound Specialist
60. Mechanical Engineer		X		X		0-19.01	M	M	M	S	M	M	Plans and Aids in Solving Mechanical Problems
61. Refrigerating Engineer		X		X		5-72.310	M	M	M	S	M	M	Plans and Aids in Solving Refrigeration Problems
62. Mining Engineer		X		X		0-20.0	M	M	M	L	M	M	Directs Removal of Mineral Deposits from Earth
63. X-Ray Technician		X		X	X	0-50.04	M	S	M	S	S	M	Assists in Taking and Developing X-Ray Pictures
64. Atomic Research Physicist	X	X		X		0-35.7	M	M	M	M	M	M	Assists in Developing Use of Atomic Energy
65. Electronics Physicist	X	X	X	X	X	0-35	M	M	M	M	S	M	Studies the Radar, Sonor, etc., Uses
66. Television Expert		X	X	X	X	0-17	M	M	M	L	M	M	Develops Types and Uses of Television Sets
67. Aviation Expert		X	X	X	X	0-19.03	M	S	M	L	S	M	Aids in Designing and Building Aircraft
68. Biophysicist	X	X		X		0-39.4	M	M	M	M	M	M	Studies Relation of Physics to Living Cells
69. Auto Body Expert			X	X	X	5-81.51	M	S	M	L	S	S	Repairs Car Bodies and Fenders

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